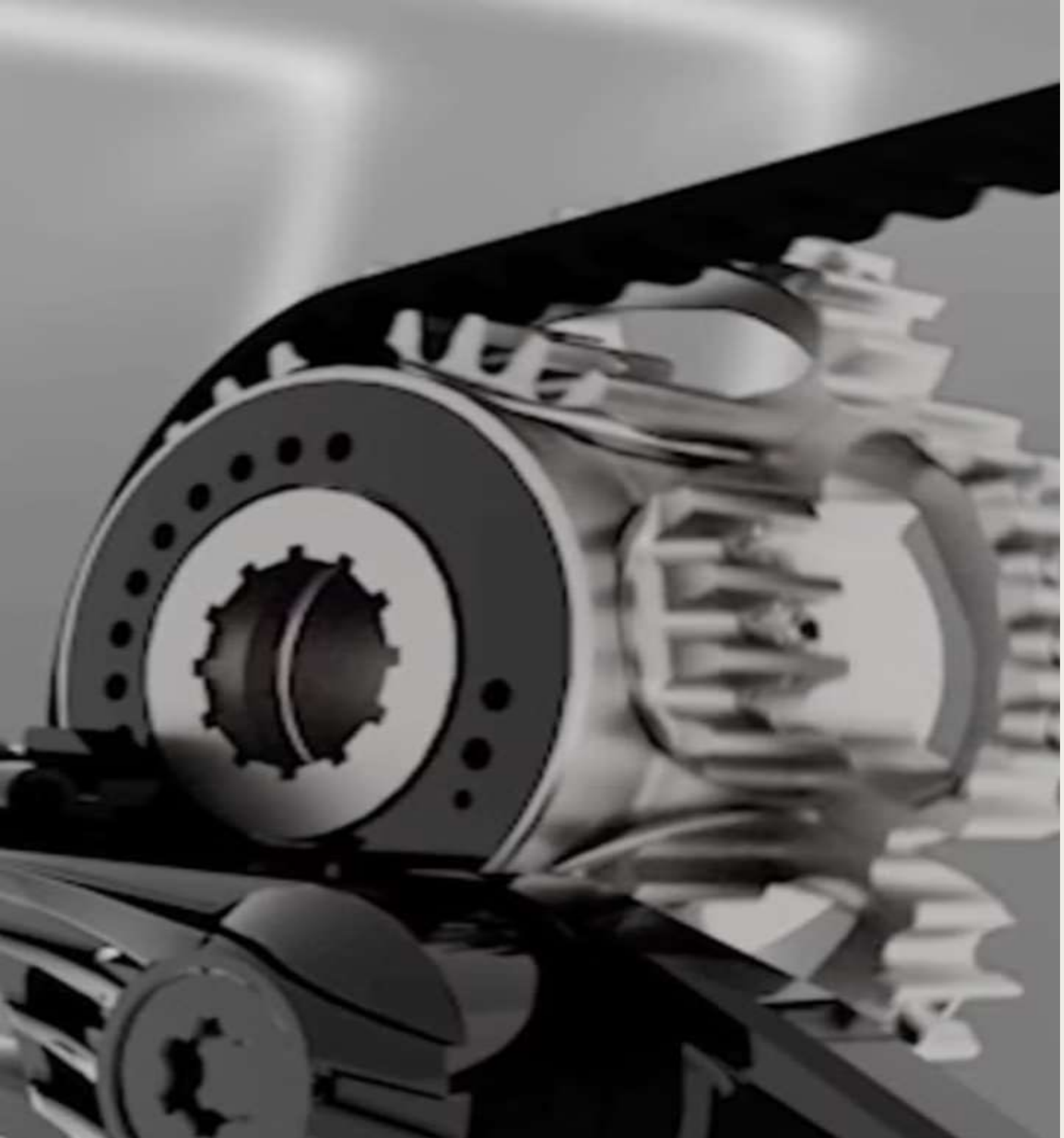

Lecture on Engineering Mechanics: Statics

Prepared by

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Lecturer
Department of ME (UGV)

BASIC COURSE INFORMATION

Course Title	Engineering Mechanics
Course Code	ME 0715-2101
Credits	03
CIE Marks	90
SEE Marks	60
Exam Hours	2 hours (Mid Exam) 3 hours (Semester Final Exam)
Level	3rd Semester

Engineering Mechanics

COURSE CODE: ME 0715-2101

CREDIT:03

Mid Exam Hours 2

TOTAL MARKS:150

Semester End Exam Hours 3

CIE MARKS: 90

SEE MARKS: 60

Course Learning Outcomes (CLOs): After completing this course successfully, the students will be able to-

CLO 1	**Define and explain** principles of engineering mechanics (example- system of forces) related to civil engineering domain.
CLO 2	**Understand** and learn relationship among physical process, kinetics and kinematics.
CLO 3	**Differentiate** concepts of principles of engineering mechanics (i.e. statics and dynamics) for different simple situations..
CLO 4	**Prepare** free body diagrams of real case phenomenon considering engineering mechanics point of view and formulate Load acting on a body, Resultant forces acting on body, direction, magnitude and position of forces acting on a body.

SL	Content of Course	Hrs	CLOs
1	Force System & Resultant Force: Types of acting Force & Force System (parallel, colinear, concurrent), Parallelogram Theory, Method of Resolution, Varignon Theorem, Law Of Triangle, Problem Solving on Parallelogram Theory & Triangle Theory, Problem Solving on Varignon & method, Problem Solving on method of resolution	6	CLO1
2	Free body diagram : Understanding forces , Drawing FBD, Applying FBD, Decomposing forces into x and y components ,Applying FBDs to complex situations with multiple objects and forces ,Using FBDs in conjunction with work and energy concepts.	6	CLO4
3	Centre of Gravity: Centroid, Methods of Centre of Gravity, Centre of Gravity by Geometrical Considerations, Axis of Reference, Centre of Gravity of Symmetrical Sections, Centre of Gravity of Unsymmetrical Sections, Centre of Gravity of Solid Bodies.	6	CLO2,CLO 3, CLO4
4	Friction: Static, Dynamic Friction, Co efficient of Friction, Law of Static and Dynamic Friction, Ladder,Wedge and Screw Friction, Problems solving on Friction.	10	CLO2,CLO 3, CLO4
5	Truss : Introduction to truss , Applying loads, Truss components, Simple truss analysis, Truss design consideration.	4	CLO2,CL O3, CLO4

Text Book:

1) A Textbook of Engineering Mechanics- R.S Khurmi

2) Vector Mechanics for Engineers (Statics)- Beer& Johnston

ASSESSMENT PATTERN

CIE- Continuous Internal Evaluation (90 Marks)

Bloom's CategoryMarks (out of 90)	Tests (45)	Assignments(10)	Class Test (20)	Quiz(5)	External Participation in Curricular/Co-CurricularActivities (10)
Remember	5		10	05	
Understand	5	05	10		
Apply	10				**10**
Analyze	15				
Evaluate	10				
Create		**05**			

SEE- Semester End Examination (60 Marks)

Bloom's Category	Test
Remember	10
Understand	10
Apply	10
Analyze	10
Evaluate	10
Create	10

Couse plan specifying content, CLOs, teaching learning and assessment strategy mapped with CLOs

Week	Topic	Teaching-Learning Strategy	Assessment Strategy	Corresponding CLOs
1	Introduction to Engineering Mechanics, Force system & principle of moment	Lecture, open Discussion, PPT	Quiz	CLO1
2	Force Components & Resultant Force	Oral Presentation, Lecture, PPT, Discussion	Written exam, Quiz, CT	CLO4
3	Force Components & Resultant Force	Oral Presentation, Lecture, PPT, Discussion	Assignment, Written, Quiz, CT	CLO2, CLO4
4	Review on Lami, Method of Resolution and Varignons Theorem	Oral Presentation, Lecture, PPT, Discussion	Assignment, Written, Quiz, CT	CLO2, CLO4
5	Problem Solving on Resultant Force	Oral Presentation, Lecture, PPT, Discussion	Assignment, Written, Quiz, CT	CLO2, CLO4
6	Types of Supports in Structure	Oral Presentation, Lecture, PPT, Discussion	Written, Quiz, CT	CLO 1, CLO4
7	Center of gravity related theory	Oral Presentation, Lecture, PPT, Discussion	Assignment, Written, Quiz	CLO2, CLO4
8	Center of gravity related Problem Solving	Oral Presentation, Lecture, PPT, Discussion	Assignment, Written, Quiz, CT	CLO4, CLO3
9	Center of gravity related problem Solving	Oral Presentation, Lecture, PPT, Discussion	Assignment, Written, Quiz, CT	CLO4, CLO3
10	Friction related theory	Oral Presentation, Lecture, PPT, Discussion	Written, Quiz, CT	CLO3

11	Friction related theory	Oral Presentation, Lecture, PPT, Discussion	Written, Quiz, CT	CLO2, CLO4
12	Friction related problems	Oral Presentation, Lecture, PPT, Discussion	Assignment, Written, Quiz, CT	CLO3
13	Friction related problems	Oral Presentation, Lecture, PPT, Discussion	Assignment, Written, Quiz, CT	CLO3
14	Ladder and Wedge Friction	Oral Presentation, Lecture, PPT, Discussion	Assignment, Written, Quiz, CT	CLO3
15	Simple Truss Problems	Oral Presentation, Lecture, PPT, Discussion	Assignment, Written, Quiz, CT	CLO2, CLO4
16	Simple Truss Problems	Oral Presentation,	Assignment, Written, Quiz, CT	CLO2, CLO3
17	Review, Practice and Exercise	Group Discussion		

Week -1

Lecture

On

Introduction to Engineering Mechanics,
Force systems, Types of Force &
Principle of moment

(10-35)

Introduction to Engineering Mechanics ,Statics

Engineering Mechanics

Engineering mechanics is the branch of engineering that applies physics and mathematical methods to analyze the behavior of physical systems subjected to forces and displacements. It encompasses various sub-disciplines, including statics, dynamics, and mechanics of materials. It's the foundation for designing structures, machines, and many other engineered systems.

Statics

Statics is the subfield of engineering mechanics that deals specifically with bodies at rest, or in a state of constant motion. It focuses on analyzing forces that are balanced, resulting in no net force or acceleration. The primary concern is understanding equilibrium conditions and ensuring structural stability.

Applications

Structures, machines, and many other engineering systems

Fundamental Concepts & Principles

1 **Force**

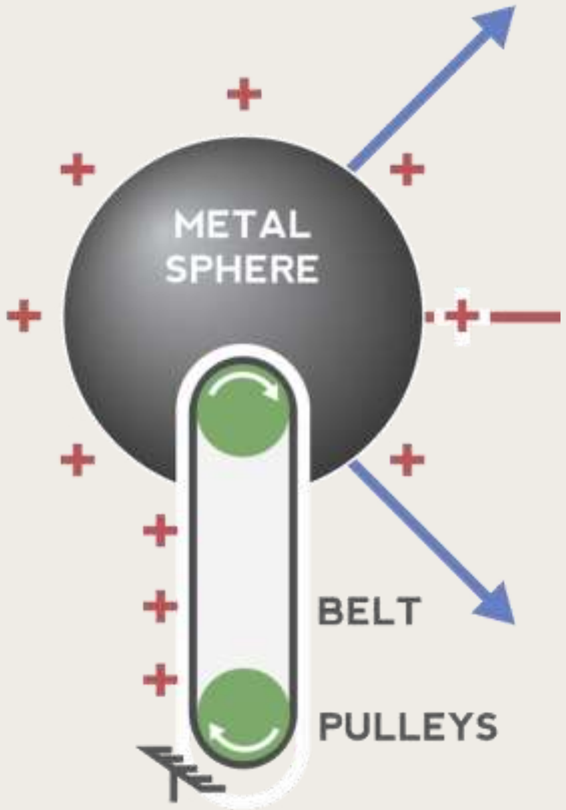
Push or pull that can cause motion or deformation

2 **Moment**

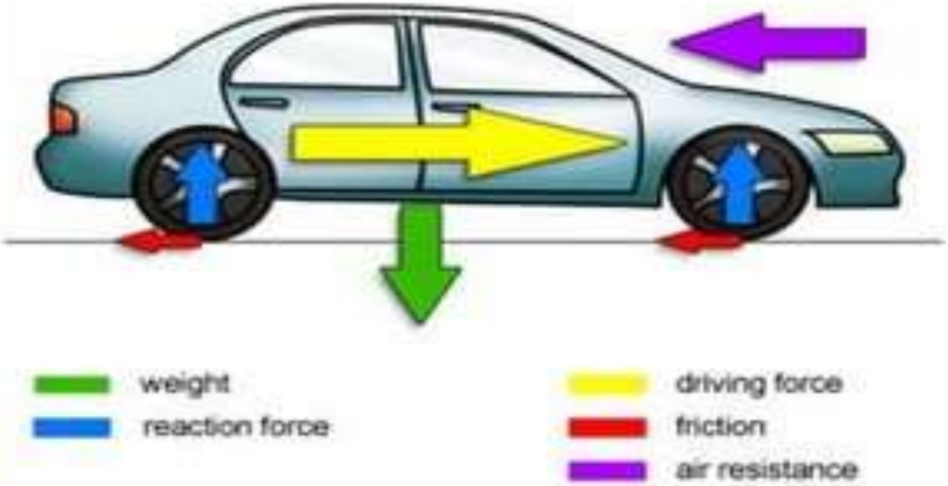
Tendency of a force to rotate an object

3 **Equilibrium**

State of balance where all forces and moments cancel out

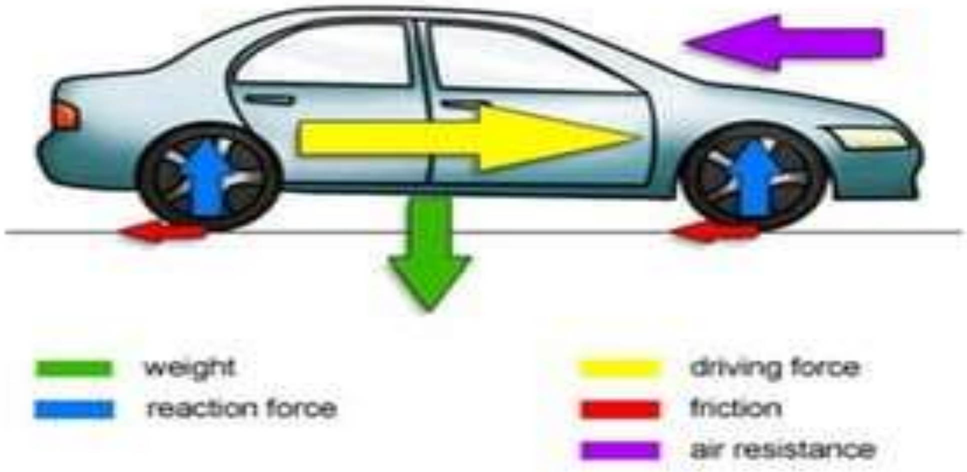


What is the need of knowing MECHANICS?



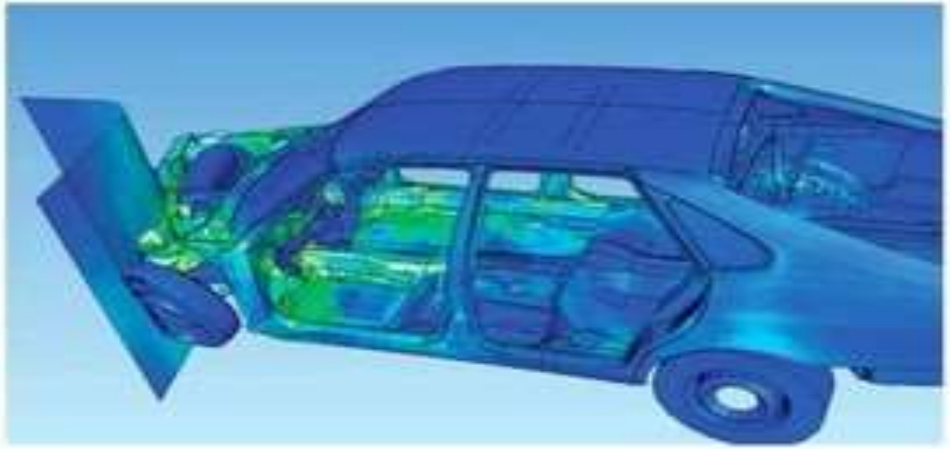
Mechanics → Deals with forces

Rigid body mechanics



Studying External effect of forces on a body such as velocity, acceleration, displacement etc.

Studying Internal effect of forces on a body such as stresses (internal resistance), change in shape etc.



Deformable body mechanics

Statics

- Deals with forces and its effects when the body is at rest

Truss Bridge

Dynamics

- Deals with forces and its effects when the body is in moving condition

IC Engine

STATICS

- It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies at rest.

DYNAMICS

- It is that branch of Engineering Mechanics, which deals with the forces and their effects, whileacting upon the bodies in motion.
- The subject of Dynamics may be further sub-divided into the
- following two branches :
- 1. **Kinetics, and 2. Kinematics.**

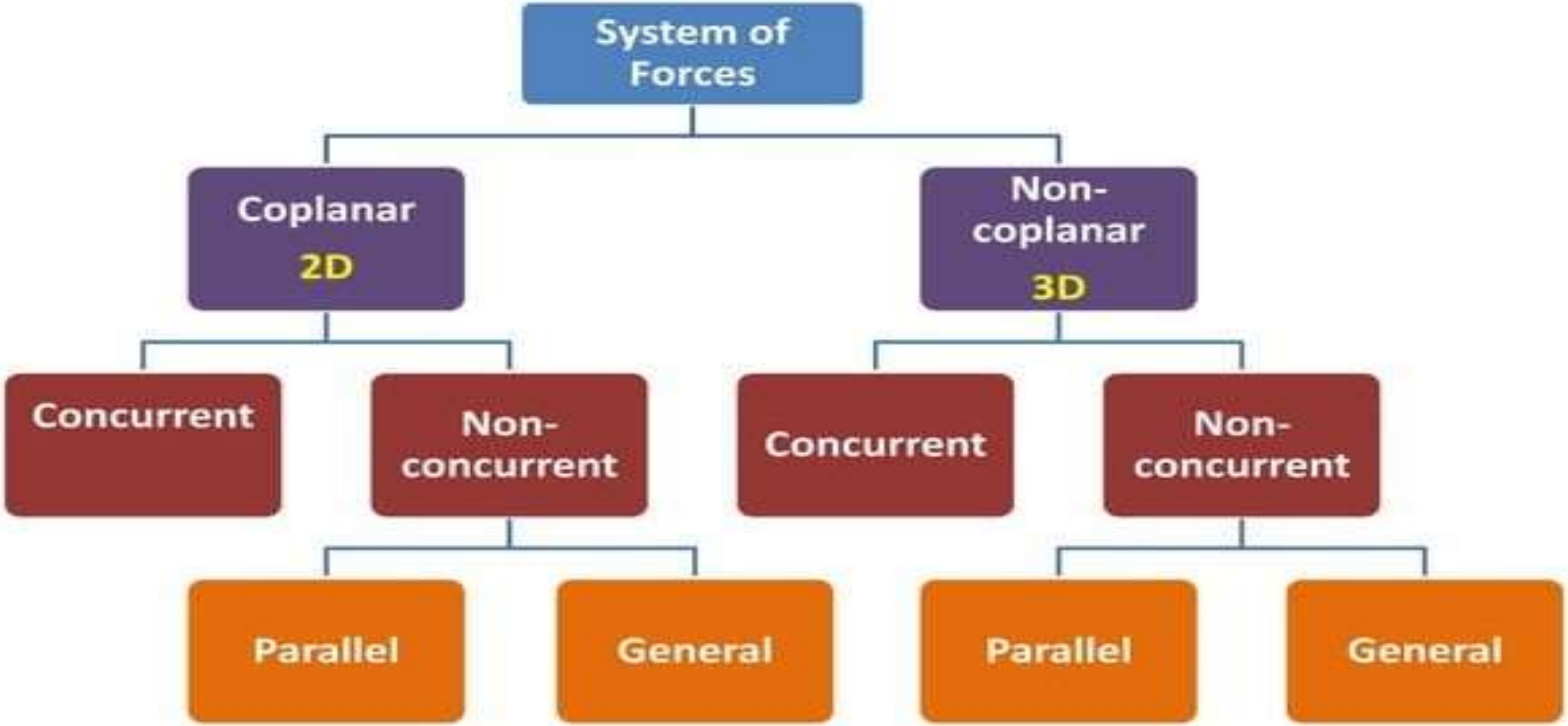
KINETICS

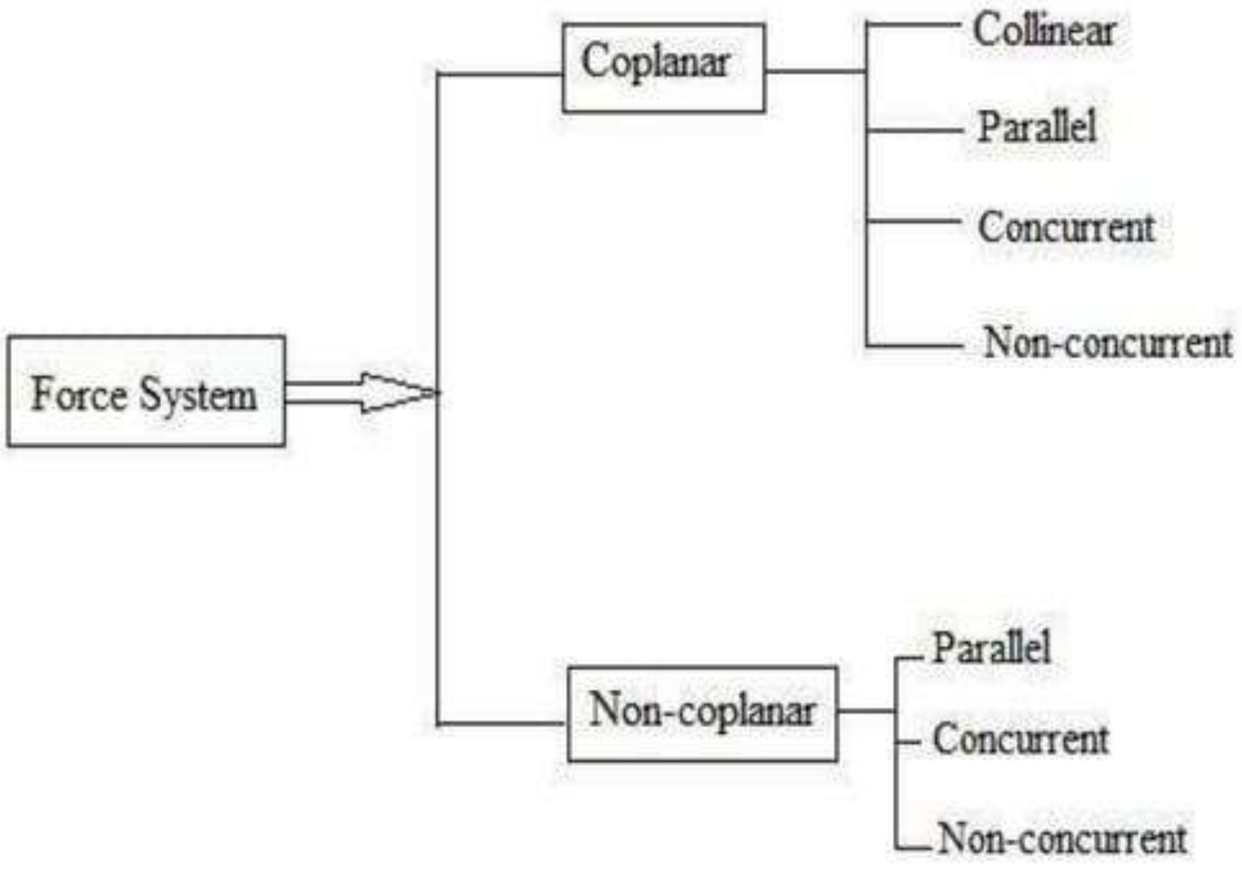
- It is the branch of Dynamics, which deals with the bodies in motion due to the applicationof forces.

KINEMATICS

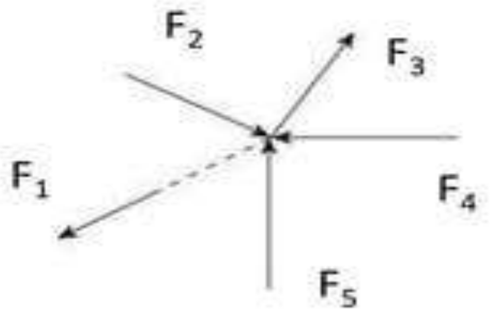
- It is that branch of Dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

System of Forces: Several forces acting simultaneously upon a body




Force System
Coplanar
Collinear
Parallel
Concurrent
Non-concurrent
Non-coplanar
Parallel
Concurrent
Non-concurrent

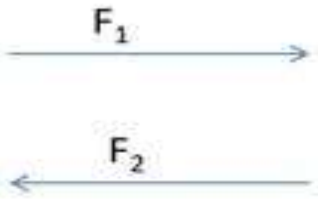
•**Coplanar forces- Coplanar forces** means the **forces** in a plane.
Or a system in which all the **forces** lie in the same plane, it is known as coplanar **force** system.



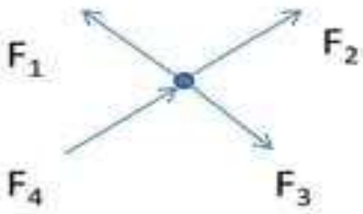
•**Coplanar Collinear forces** - **Collinear forces** are those **forces** which have a common line of action, i.e. the line of action of the **forces** lie along a single straight line either they are push or pull in nature.
Examples: two people standing at the opposite ends of a rope and pulling on it.

Pull forces acting in a same line or same line of action

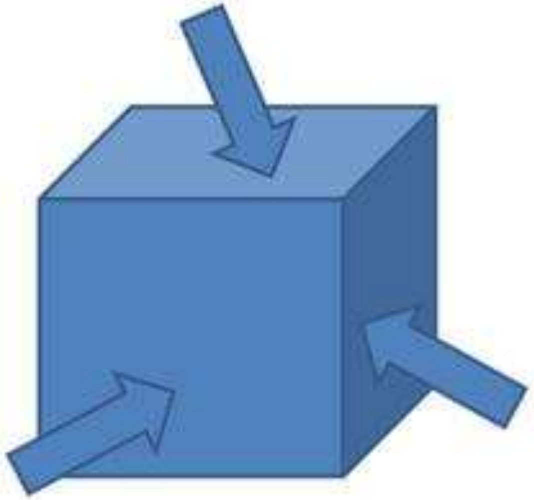
•**Coplanar Parallel forces**- Parallel forces are those forces which are in the same plane but never intersect by each other and they may be same or opposite in direction.



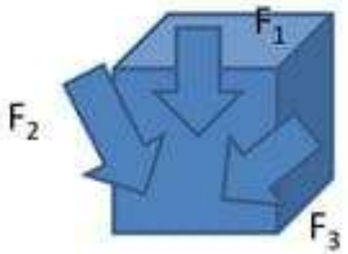
•**Coplanar Concurrent Forces-** Concurrent forces are those forces which are acting at a same point and at a same plane, also they may be pull or push in nature.



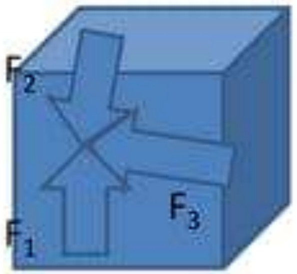
Non-coplanar forces-Non-coplanar forces are those forces which are not acting from a same plane.



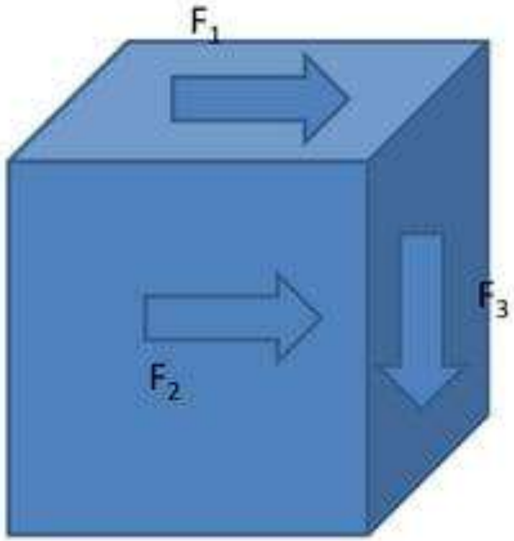
•Non coplanar non- concurrent forces- Non- Coplanar non-concurrent forces are those forces which are not acting at a same point and not at a same plane, also they may be pull or push in nature.



•Non-coplanar concurrent forces- Non- Coplanar concurrent forces are those forces which are acting at a same point but not from a same plane, also they may be pull or push in nature.



•Non-coplanar parallel forces- Non-coplanar Parallel forces are those forces which are not in the same plane and never intersect by each other, they may be same or opposite in direction.

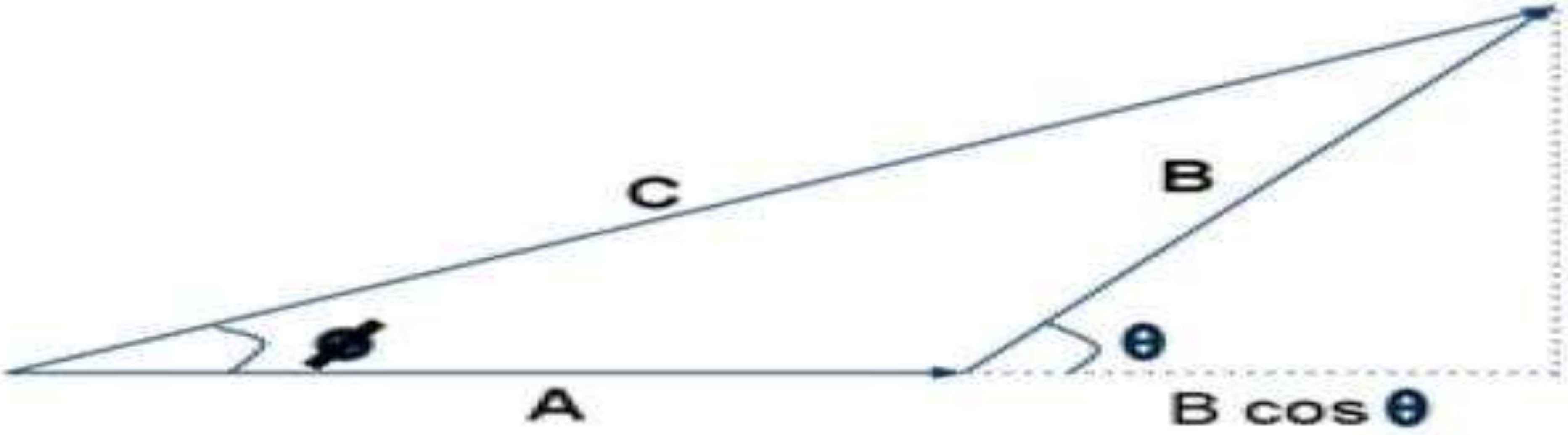


Triangle Law of Vectors

- Triangle Law of Vectors states that if two vectors are represented as adjacent sides of a triangle then the third side taken in opposite order is the resultant of the two. This law is used to find the resultant of two vector which gives both magnitude and direction

Step: 3

To find the magnitude and direction of the resultant,



'θ' is the angle between $\vec{A}$ & $\vec{B}$.

'ø' is the angle between $\vec{C}$ & $\vec{A}$.

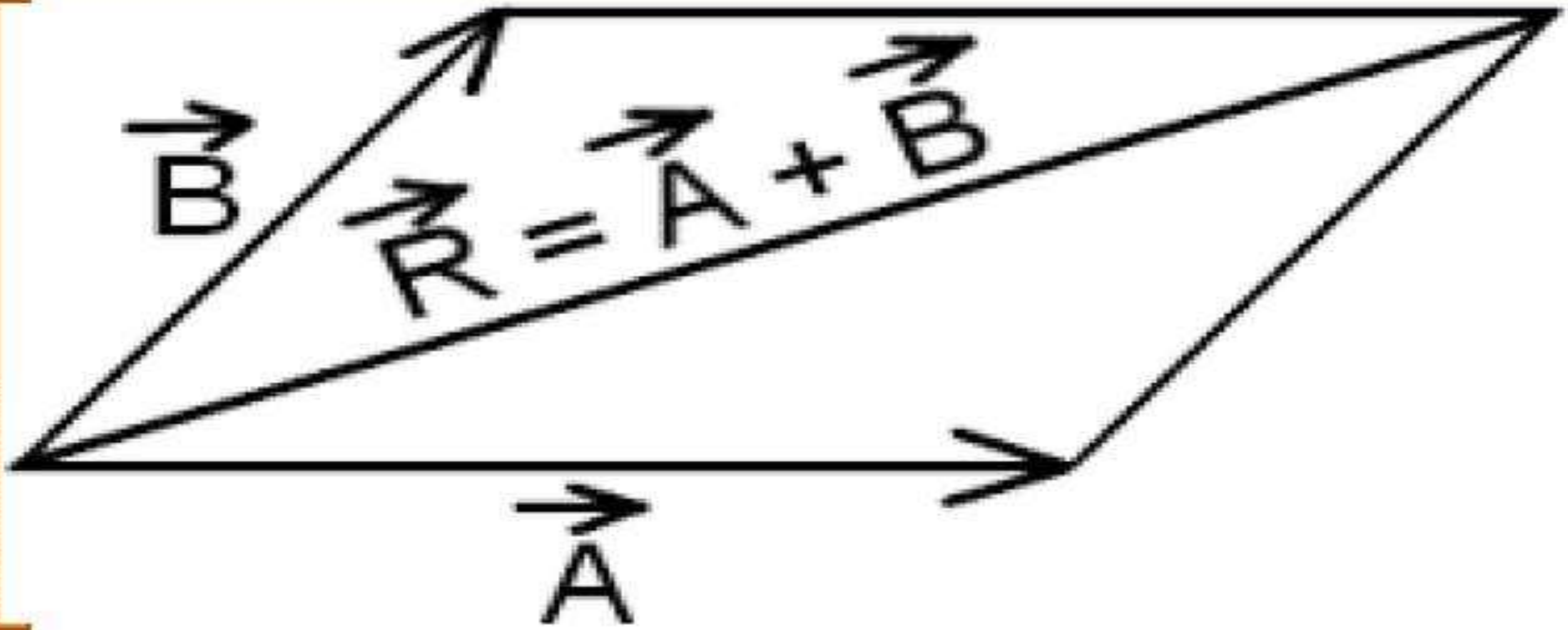
$$|\vec{C}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\phi = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

$\vec{C}$ makes an angle ø with the vector $\vec{A}$.

Parallelogram Law of Forces

- If two forces, acting at a point, are represented in magnitude and direction by the two sides of a parallelogram drawn from one of its angular points, their resultant is represented both in magnitude and direction by the diagonal of the parallelogram passing through that angular point.


B
R = A + B
A

$$F_R = \sqrt{(F_1)^2 + (F_2)^2 + 2F_1F_2 \cos\theta}$$

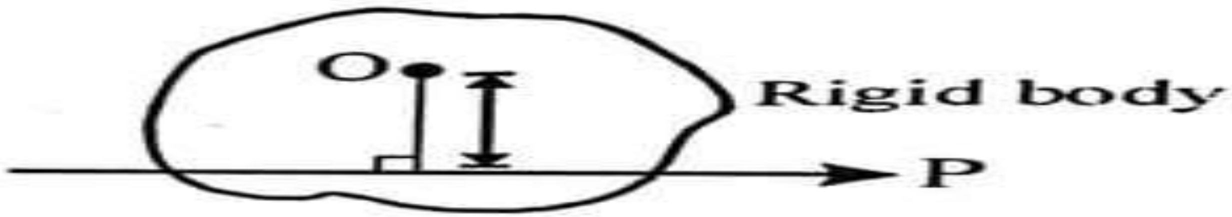
$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

It the angle (α) which the resultant force makes with the other force F_2.

$$\tan \alpha = \frac{F_1 \sin \theta}{F_2 + F_1 \cos \theta}$$

Moment of Force:

Moment of a force is the rotation or turning effect produced by a force.



Moment= (Force) x (Perpendicular from Point)

Mathematically,

M_o= P x d

Unit of Moment =Nm or kN-m

Types of Moment

1. Clockwise moment (taken as positive)
2. Anticlockwise moment (taken as Negative)

Varignon's Theorem:

"The algebraic summation of moments of all forces acting on a rigid body about any point is equal to the moment of their resultant force about same point".

Let, ΣM_A = Algebraic summation of moments of all forces about a point A

R= Resultant Force for a given force system

d= perpendicular distance of line of action of R from point A

Mathematically, ΣM_A= R x d

$$d = \frac{\Sigma MA}{R}$$

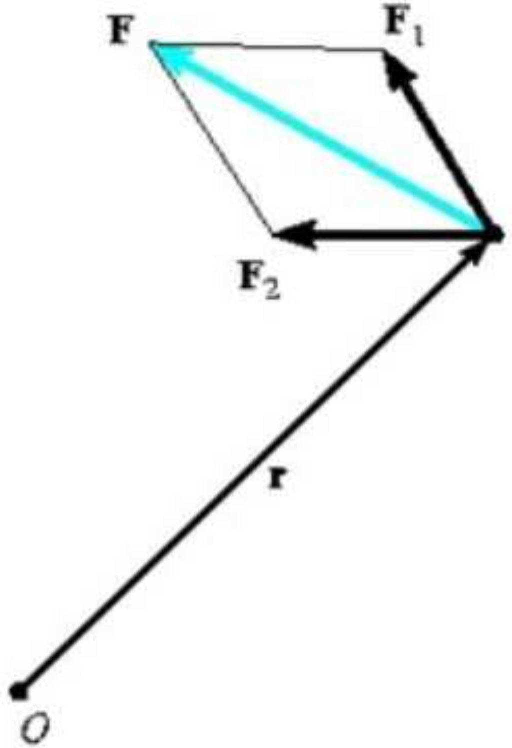
4.4 Principles of Moments

- Also known as Varignon's Theorem

  "Moment of a force about a point is equal to the sum of the moments of the forces' components about the point"

- For $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$,

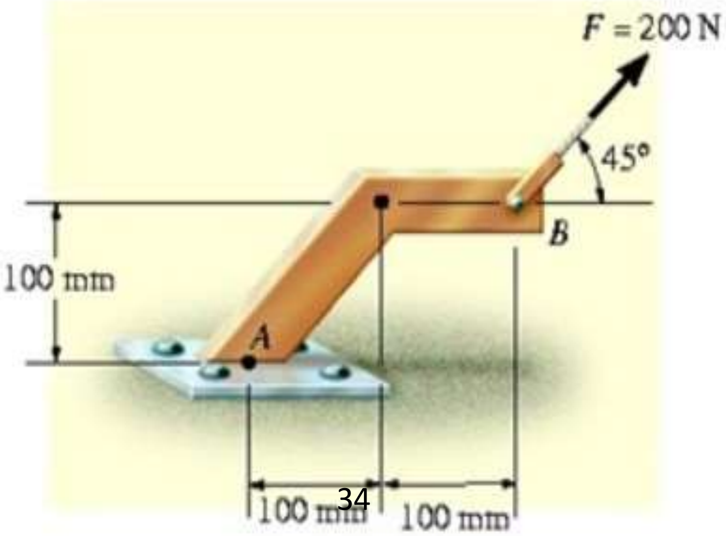
  $$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$
  $$= \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2)$$
  $$= \mathbf{r} \times \mathbf{F}$$



4.4 Principles of Moments

Example 4.6

A 200N force acts on the bracket. Determine the moment of the force about point A.



(a)

4.4 Principles of Moments

Solution

Method 2:

- Resolve 200N force into x and y components
- Principle of Moments

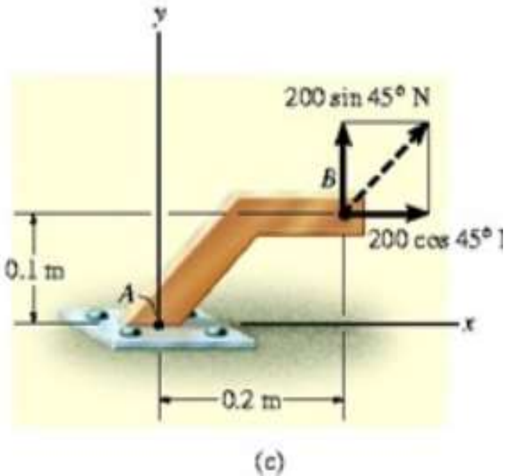
$$M_A = \sum Fd$$

$$M_A = (200\sin 45^\circ N)(0.20m) - (200\cos 45^\circ)(0.10m)$$

$$= 14.1 \text{ N.m (CCW)}$$

Thus,

$$\mathbf{M}_A = \{14.1\mathbf{k}\}\text{N.m}$$



Week -2

Lecture

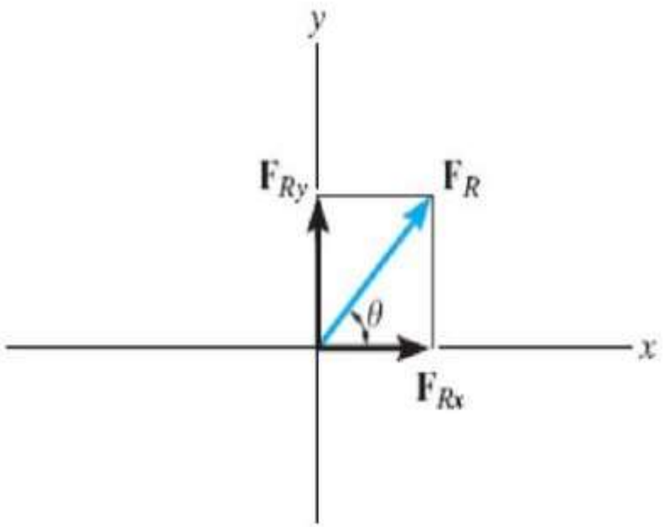
On

Force Components & Resultant Force

(36-53)

Force Components & Resultants

$$F_{Rx} = \sum F_x$$
$$F_{Ry} = \sum F_y$$

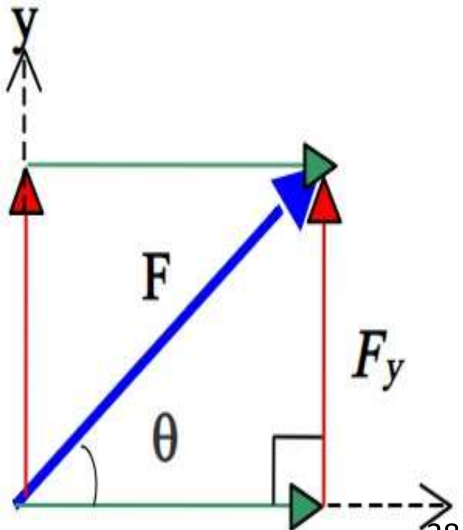


Magnitude of $\mathbf{F}_R$ can be found by Pythagorean Theorem

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \quad \text{and } \theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

2.7 Resolution of A Force Into Perpendicular Components

One of the most important and useful concept is the resolution of a force vector into two perpendicular components, in the **x and y axis** as denoted by the F_x (x) *and* F_y (y) components.

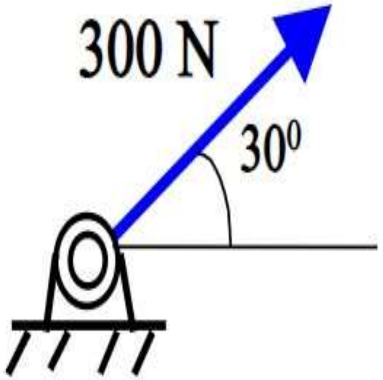


$$F_x = F \cos \theta$$

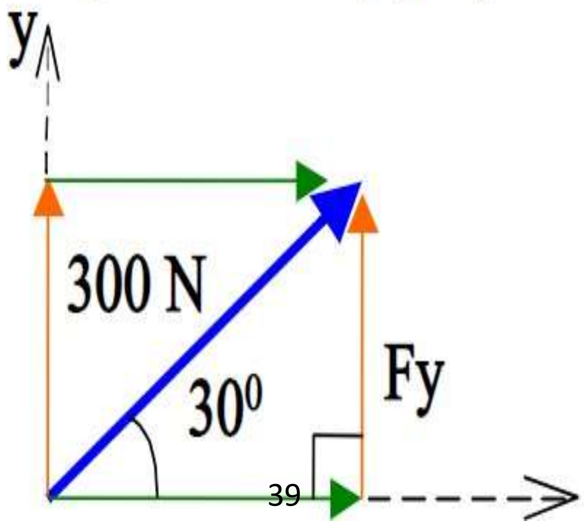
$$F_y = F \sin \theta$$

Example 2.1

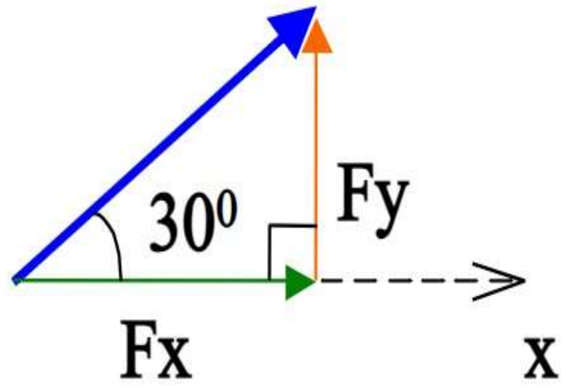
Resolve the 300 N force into components along the x and y axes.



Sketch the parallelogram with Fy perpendicular to Fx.



Select any half of the triangle, a vector triangle, for analysis.



As this is a right-angle triangle,

$$\cos 30^0 = Fx / 300$$

i.e $$Fx = 300 \times \cos 30^0$$

$$= 259.81 \text{ N}$$

Also since $\sin 30^0 = Fy / 300$

i.e $$Fy = 300 \sin 30^0$$

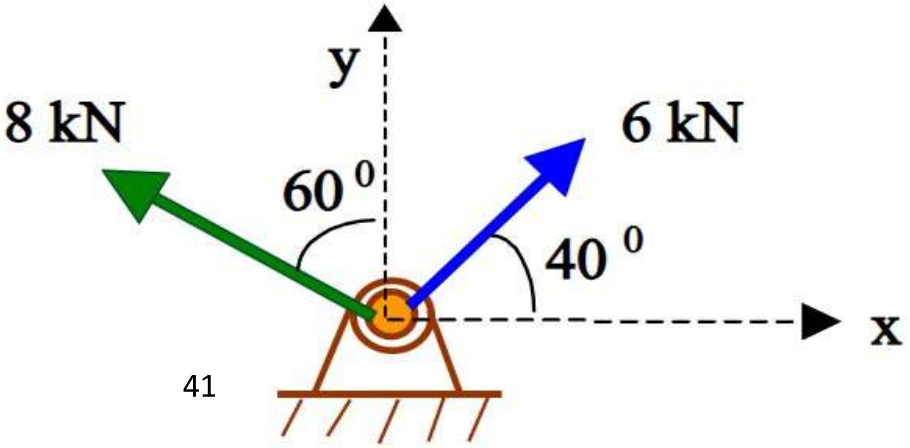
$$= 150 \text{ N}$$

2.8 Resolution of A Force By Fx And Fy Components

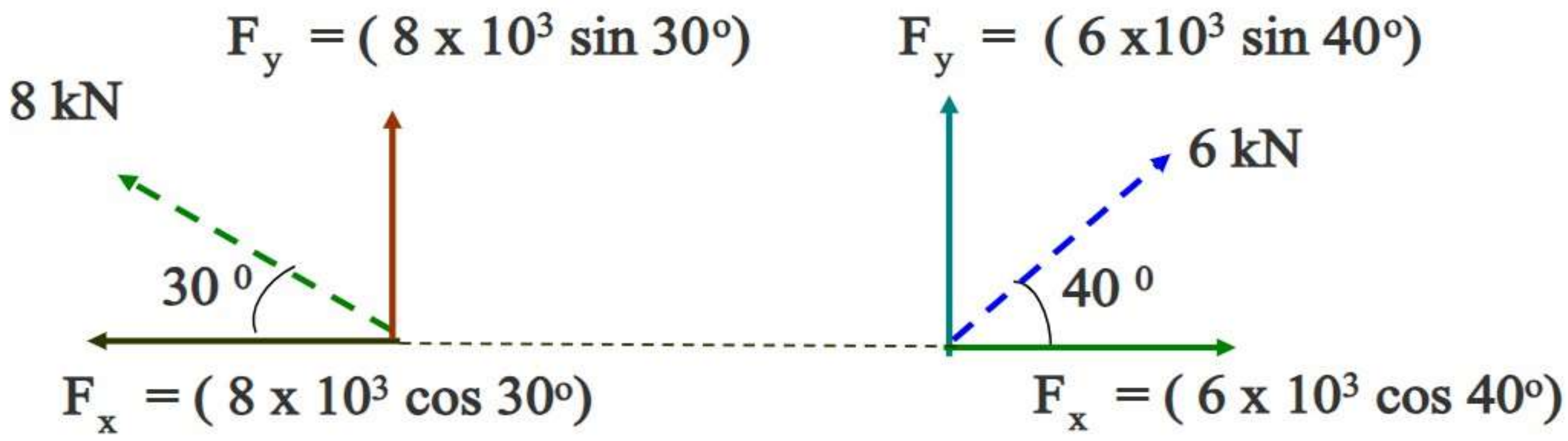
For a number of forces, this section introduces an easier analytical method of finding the resultant.

Students should master this method as it would form the basis of all your calculations for resultant forces.

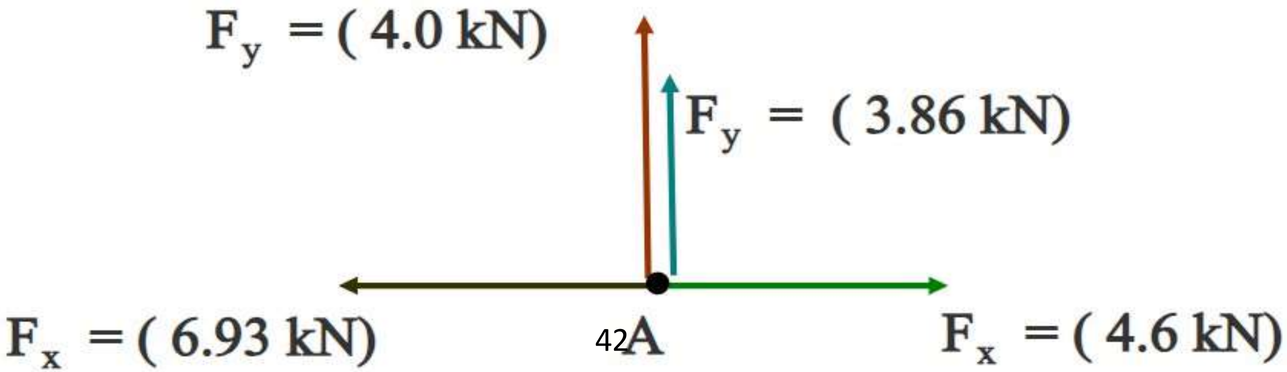
Let us use the same simple example in section 2.3 above to demonstrate the method:



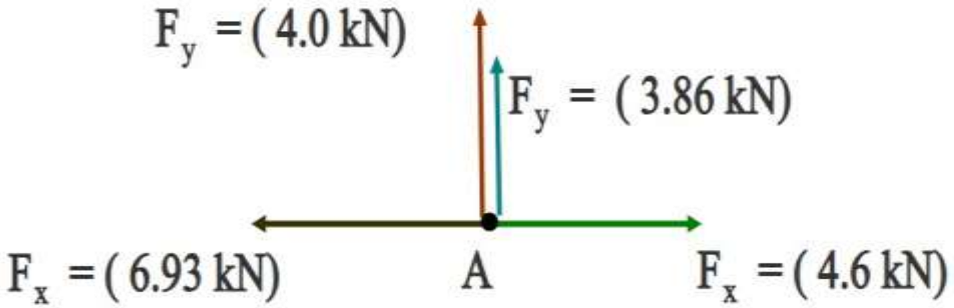
1. *We first resolved the 2 forces into perpendicular i.e. Fx and Fy components.*



2. *The 2 forces can now be replaced by the above 4 components as shown below.*



3. *Now we can add the two Fx forces to form a single Rx, as they are both horizontal forces but only opposite in direction, if we adopt a sign convention. Similarly, we can also add the two Fy forces to form a single Ry.*



ΣFx = Rx, is the sum of all the Fx forces ,

ΣFy = Ry , is the sum of all Fy forces .

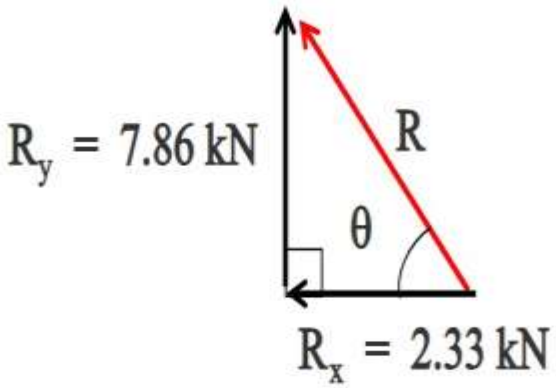
$\Sigma Fx = (4.6x10^3) - (6.93x10^3)$ {Fx pointing left is –ve}

Rx = - 2.33 kN ←

$\Sigma Fy = (3.86x10^3) + (4.0x10^3)$ {Fy pointing up is +ve}

Ry = + 7.86 kN ↑

4. These two forces, Rx and Ry can now replace the previous four component forces in (2) above, and we can use the vector triangle method to add them back to form a single force, which is the resultant R, as shown by the vector triangle below.



From the right-angle triangle, we then use the Pythagoras Theorem to find the magnitude of R.

The magnitude of R is:

$R = \sqrt{\{R_x^2 + R_y^2\}}$

$= \sqrt{\{(2.33x10^3)^2 + (7.86x10^3)^2\}}$

$= 8.2\ kN$

The angle θ of R is:

$\theta = \tan^{-1}(Ry/Rx)$

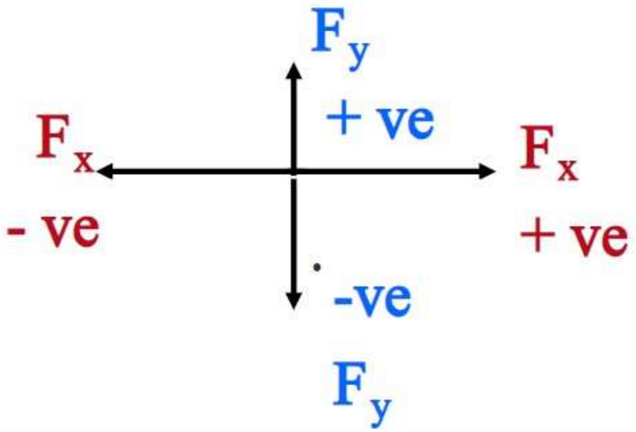
$= \tan^{-1}((7.86x10^3)/(2.33x10^3))$

$\theta = 73.5^0$

Same answers as before .

Resultant By Method of Perpendicular Components:

Use the ***Sign convention*** *for the force components:*

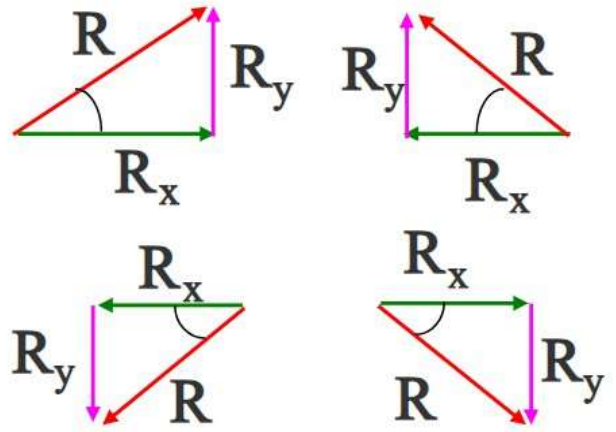


1. *Resolve all (but not horizontal or vertical) forces into Fx and Fy components.*

2. *Use* $\Sigma F_x = Rx,$ $\Sigma F_y = Ry$

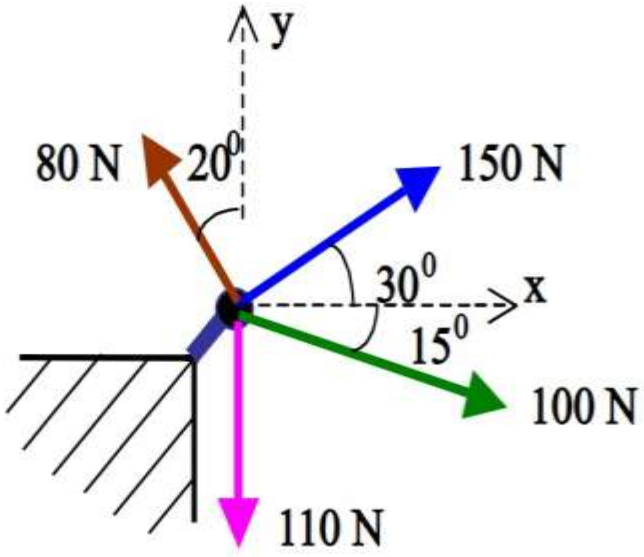
3. *Magnitude:* $R = \sqrt{\{R_x^2 + R_y^2\}}$
   Angle: $\theta = \tan^{-1}(Ry/Rx)$
   (from x axis)

4. Direction of R:



Example 2.2

Four forces act on a bolt as shown. Determine the resultant force.



Solution :

Fx components in the x direction

$F_{1x} = +150 \cos 30^0 = 129.9$ N

$F_{2x} = +100 \cos 15^0 = 96.6$ N

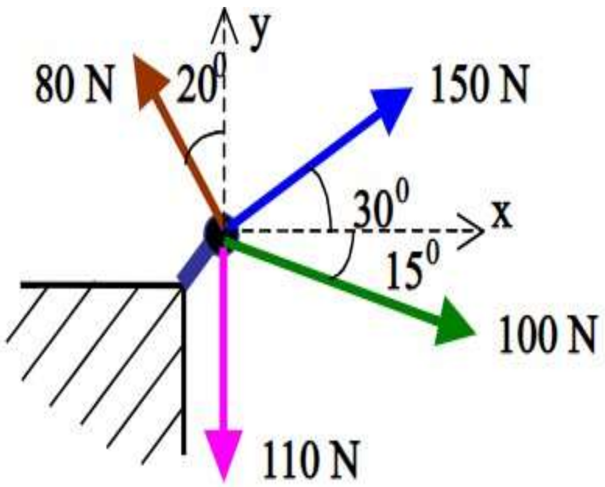
$F_{3x} = -80 \cos 70^0 = -27.4$ N

$F_{4x} = 0$ N, (for 110 N force)

i.e. $\sum F_x = R_x = 150 \cos 30^0 + 100 \cos 15^0 - 80 \cos 70^0$

$= 129.9 + 96.6 - 27.4 + 0$

$R_x = 199.1\text{N} \longrightarrow$



Fy components in the y direction

$F_{1y} = +150 \sin 30^0 = 75$ N

$F_{2y} = -100 \sin 15^0 = -25.9$ N

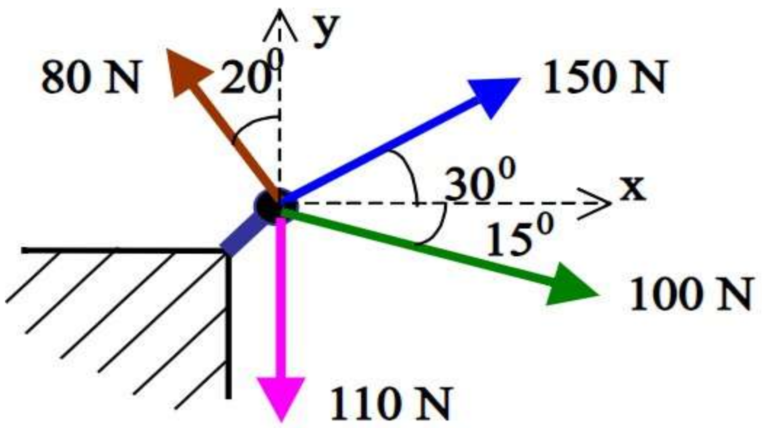
$F_{3y} = +80 \sin 70^0 = 75.2$ N

$F_{4y} = -110$ N (it is a Fy or vertical force)

i.e. $\Sigma F_y = R_y = 150 \sin 30^0 - 100 \sin 15^0 + 80 \sin 70^0$

$= 75 - 25.9 + 75.2 - 110$

$R_y = 14.3$ N ↑



The magnitude of R is

$$R = \sqrt{(R_x^2 + R_y^2)}$$

$$R = \sqrt{(199.1^2 + 14.3^2)}$$

$$= 199.6 \text{ N}$$

The angle θ, R makes with the horizontal is

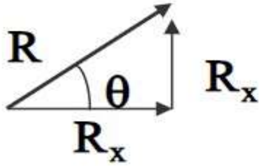
$$\theta = \tan^{-1} R_y / R_x$$

$$= \tan^{-1} 14.3 / 199.1$$

$$= 4.1^0$$

Since R_x ⟶ and R_y ↑ are positive values,

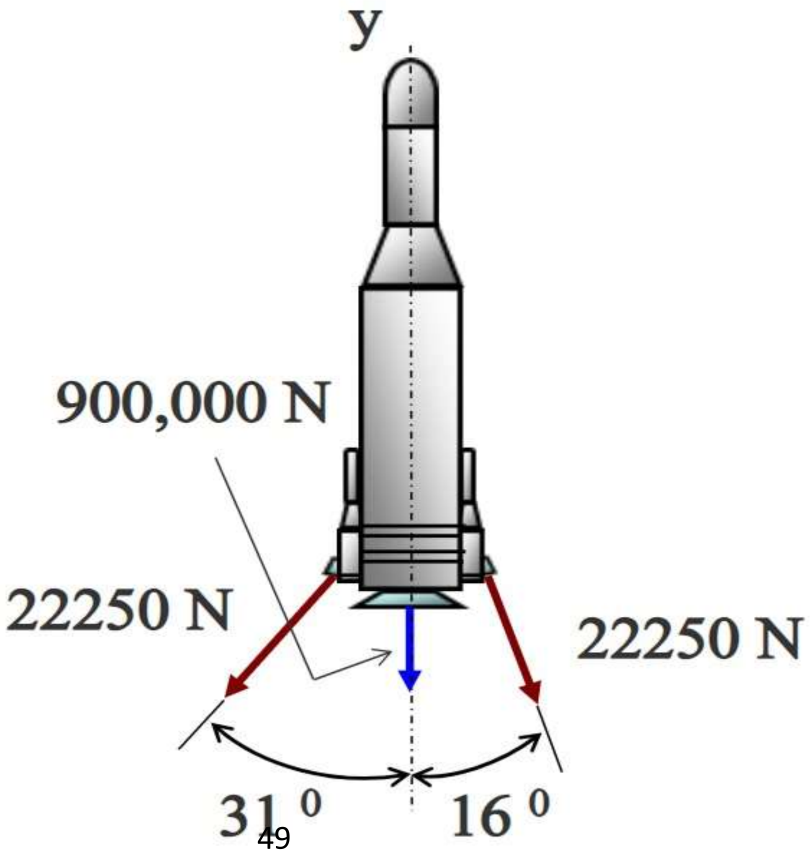
$R = 199.6$ N and $\theta = 4.1^0$



Example 2.3

The rocket's main engine exerts a total thrust of 900,000 N parallel to the y axis. Each of its two small side engines exerts a thrust of 22,250 N in the direction as shown.

Determine the magnitude and direction of the resultant force exerted on the rocket by the engines.



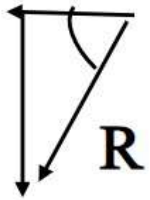
Solution:

$$\Sigma Fx = Rx = 22{,}250 \sin 16 - 22{,}250 \sin 31$$
$$= -5326.67\ N \leftarrow$$

$$\Sigma Fy = Ry = -22{,}250 \cos 16 - 22{,}250 \cos 31 - 900{,}000$$
$$= -21388 - 19072 - 900{,}000$$
$$= -940460\ N \downarrow$$

$$R = \sqrt{(Rx^2 + Ry^2)} = \sqrt{(5326.67^2 + 940460^2)} = 940475\ N$$

$$\tan \theta = Ry / Rx = 940460 / 5326.67 = 176.55$$
$$\theta = 89.68^0$$



Special situations:

1. Resultant's direction is along the x-axis (Horizontal)

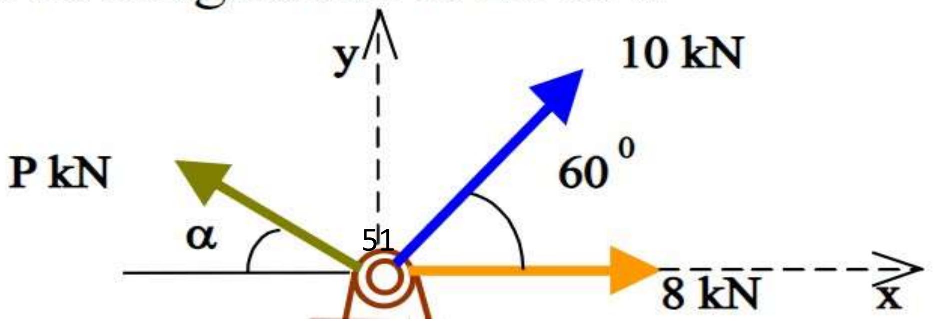
$$\Sigma F_y = R_y = 0 \qquad \Sigma F_x = R_x = R$$

2. Resultant's direction is along the y-axis (Vertical)

$$\Sigma F_x = R_x = 0 \qquad \Sigma F_y = R_y = R$$

Example 2.4

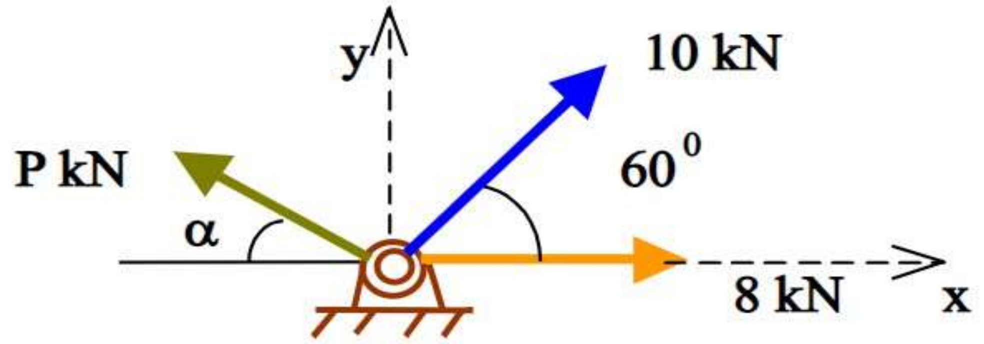
A bracket is being pulled by 3 forces as shown. Determine the angle α and the magnitude of force P if the resultant's direction is along the y-axis and its magnitude is 20 kN.



Since the *resultant's direction is along the y-axis* .

i.e $\Sigma F_x = R_x = 0$ (No R_x component)

and $\Sigma F_y = R_y = 20\text{x}10^3$ N (R must be = R_y)



$\Sigma F_x = R_x = 0$

$(10\text{x}10^3)(\cos 60^o) + (8\text{x}10^3) - (P\text{x}10^3)(\cos \alpha) = 0$ (1)

and $\Sigma F_y = R_y = 20\text{x}10^3$ N

$(10\text{x}10^3)(\sin 60^o) + (P\text{x}10^3)(\sin \alpha) = 20$ (2)

From equation (1), $P \cos \alpha = 13$ (3)

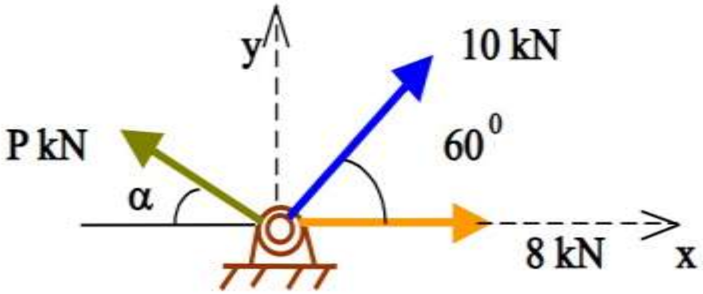
From equation (2), $P \sin \alpha = 11.34$ (4)

Equation (4)/(3), $P \sin \alpha / P \cos \alpha = (11.34 / 13)$

i.e. $\tan \alpha = 11.34 / 13 = 0.8723$

$\alpha = \tan^{-1} (0.8723)$

$\alpha = 41.1^0$



To find magnitude of P, substitute $\alpha = 41.1^0$ into equations (1) or (2).

$P \cos \alpha = 13$

$P = 13 / \cos \alpha$ = 17.25 (P kN = 17.25 kN)

End of Chapter 2

Week -3

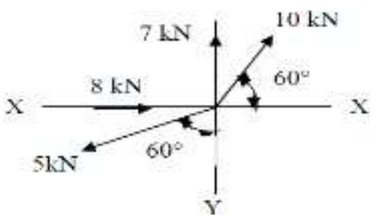
Lecture

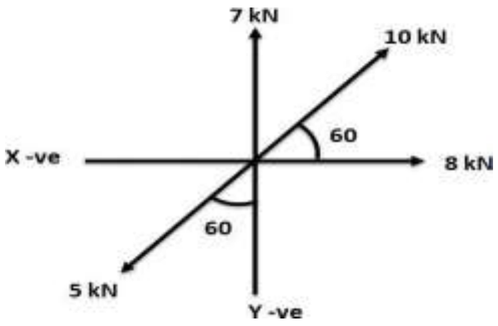
On

Force Components & Resultant Force

(54-61)

Example 1: - Find resultant of a force system shown in Figure





Answer:

1) **Given Data**

P1 = 8 kN
P2 = 10 kN
P3 = 7 kN
P4 = 5 kN

$\theta_1 = 0$

$\theta_2 = 60$

$\theta_3 = 90$

$\theta_4 = 270 - 60 = 210$

2) **Summation of horizontal force**

$$\sum H = P_1\cos\theta_1 + P_2\cos\theta_2 + P_3\cos\theta_3 + P_4\cos\theta_4 = 8.67kN(\rightarrow)$$

3) **Summation of vertical force**

$$\sum V = P_1\sin\theta_1 + P_2\sin\theta_2 + P_3\sin\theta_3 + P_4\sin\theta_4 = 13.16kN(\uparrow)$$

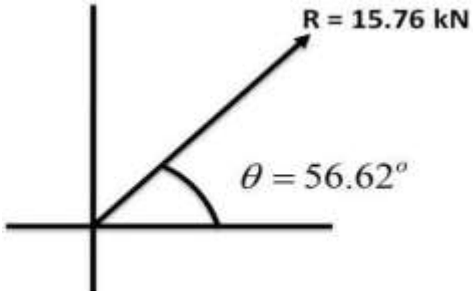
4) **Resultant force**

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2} = 15.76kN$$

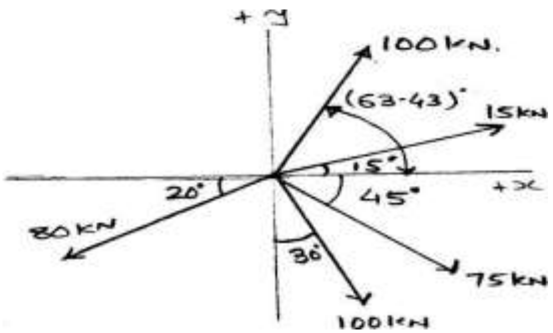
5) **Angle of resultant**

$$\tan\theta = \left|\frac{\sum V}{\sum H}\right| = 1.518$$

$$\theta = 56.62$$



Example 2 Find magnitude and direction of resultant for a concurrent force system shown in Figure



Answer

1) **Summation of horizontal force**

$\rightarrow$ **(+Ve)** $\leftarrow$ **(-Ve)**

$\sum H = +15\,\text{Cos}\,15^o + 100\text{Cos}\,(63.43)^o - 80\text{Cos}20^o + 100\text{Sin}30^o + 75\text{Cos}45^o = +87.08\,\text{kN}\,(\rightarrow)$

2) **Summation of vertical force**

$\uparrow$ **(+Ve)** $\downarrow$ **(-Ve)**

$\sum V = +15\,\text{Sin}\,15^o + 100\text{Sin}\,(63.43)^o - 80\text{Sin}20^o + 100\text{Cos}30^o + 75\text{Sin}45^o = -73.68\,\text{kN}\,(\downarrow)$

3) **Resultant force**

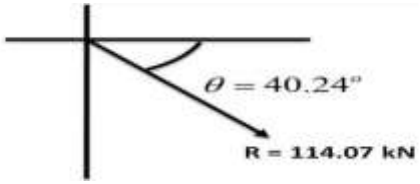
$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2} = 114.07kN$$

4) **Angle of resultant**

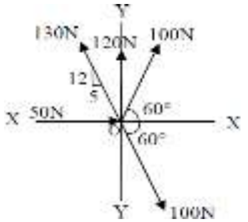
$$\tan\theta = \left|\frac{\sum V}{\sum H}\right| = 0.846$$

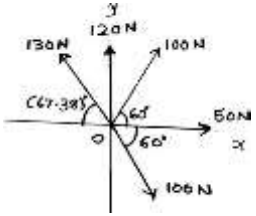
$$\theta = 40.24$$

5) **Angle of resultant with respect to positive x – axis**



Example 3 Determine magnitude and direction of resultant force of the force system shown in fig.





Answer

$$\tan\beta = \frac{12}{5} = 2.4 \qquad \therefore \beta = 67.38^0$$

1) **Summation of horizontal force**

$\rightarrow$ **(+Ve)** $\leftarrow$ **(-Ve)**

$$\sum H = +50 + 100\, \text{Cos}\, 60^\circ - 130\text{Cos}\,(67.38)^\circ + 100\text{Cos}30^\circ + 100\text{Cos}60^\circ = +100\, \text{N}\, (\rightarrow)$$

2) **Summation of vertical force**

$\uparrow$ **(+Ve)** $\downarrow$ **(-Ve)**

$$\sum V = +100\text{Sin}60^\circ + 120 + 130\text{Sin}\,(67.38)^\circ - 100\text{Sin}60^\circ - 100\text{Sin}60^\circ = +240\, \text{N}\, (\uparrow)$$

3) **Resultant force**

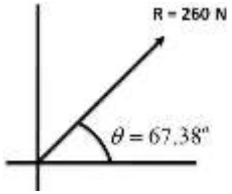
$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2} = 260N$$

4) **Angle of resultant**

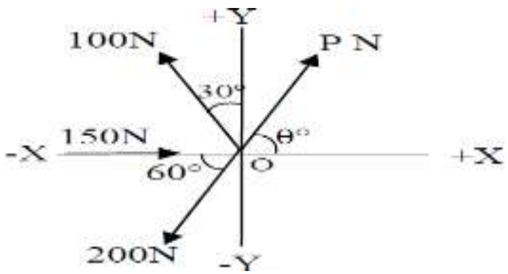
$$\tan\theta = \left|\frac{\sum V}{\sum H}\right| = 2.4$$

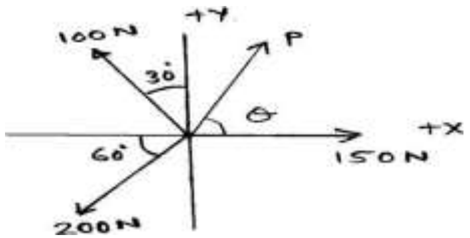
$$\theta = 67.38^\circ$$

5) **Angle of resultant with respect to positive x – axis**



Example: 4 A system of four forces shown in Fig. has resultant 50 kN along + X - axis. Determine mag- nitude and inclination of unknown force P.





Answer

As the R= 50N & directed along + X – axis.

$$\sum H = +50N \text{ and } \sum V = 0N$$

Now, $\sum H = +150 + P \cos\theta - 100 \sin 30° - 200 \cos 60° = 50$ N

$$\therefore P \cos\theta = 50 \_\_\_\_\_\_\_\_\_\_ (1)$$

Now, $\sum V = +P \sin\theta - 100 \cos 30° - 200 \sin 60° = 0$

$$\therefore P \sin\theta = 86.60 \_\_\_\_\_\_\_\_\_\_ (2)$$

From Equation (1) & (2).

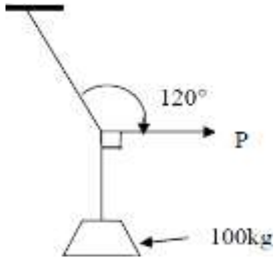
$$\tan\theta = \frac{86.60}{50}$$

$$\tan\theta = 1.732$$

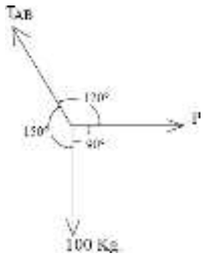
$$\therefore \theta = 60°$$

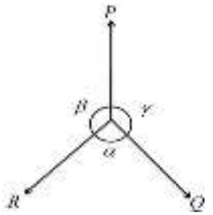
$$\therefore P = 100 \text{ N}$$

Example: 5 Find the magnitude of the force P, required to keep the 100 kg mass in the position by strings as shown in the Figure



Answer:





Free Body Diagram will be as show in fig. and there are three coplanar concurrent forces which are in equilibrium so we can apply the lami"s theorem.

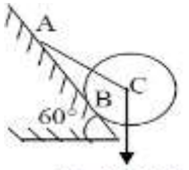
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

$$\therefore \frac{P}{\text{Sin } 150} = \frac{TAB}{\text{Sin } 90} = \frac{100}{\text{Sin } 120}$$

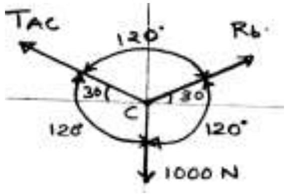
$$P = 566.38 \text{ N}$$

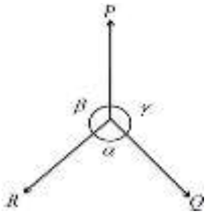
$$T_{AB} = 1132.76 \text{ N}$$

Example: 6 A cylindrical roller 600mm diameter and weighing 1000 N is resting on a smooth inclined surface, tied firmly by a rope AC of length 600mm as shown in fig. Find tension in rope and reaction at B



Answer:





Free Body Diagram will be as show in fig. and there are three coplanar concurrent forces which are in equilibrium so we can apply the lami"s theorem.

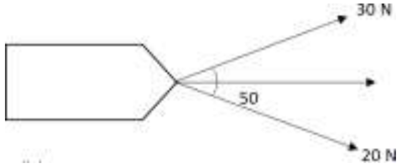
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

$$\therefore \frac{\text{Tac}}{\text{Sin } 120} = \frac{\text{Rb}}{\text{Sin } 120} = \frac{1000}{\text{Sin } 120}$$

$$\text{Tac} = 1000 \text{ N}$$

$$R_B = 1000 \text{ N}$$

Example: 7 A boat kept in position by two ropes as shown in figure. Find the drag force on the boat.



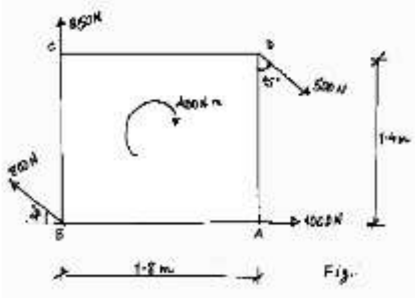
Answer:

According to law of parallelogram

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{20^2 + 30^2 + 2\times 20\times 30\cos 50} = 45.51N$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{30\sin 50}{20 + 30\cos 50} \qquad \therefore \alpha = 30.32^0$$

Example: 8 For a coplanar, non-concurrent force system shown in Fig. determine magnitude, direction and position with reference to point A of resultant force.



Answer

To find out magnitude & direction of R

Summation of horizontal force

$$\Sigma H = +500 \text{ Sin } 45^\circ - 800\text{Cos } 30^\circ + 1000 = +660.73 \text{ N } (\rightarrow)$$

Summation of vertical force

$$\sum V = -500 \text{ Cos}45^\circ + 850 + 800\text{Sin}30^\circ = +896.45 \text{ N} \quad (\uparrow)$$

Resultant force

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2} = \sqrt{(660.73)^2 + (896.45)^2} = 1113.64\ N$$

Angle of resultant

$$\tan\theta = \frac{896.45}{660.73}$$

$$\therefore \ \theta = 53.61^\circ$$

Here, we have to also locate the „R‟ @ pt. A Let the „R‟ is located at a dist[n] x from A in the horizontal direction.

Now this dist[n] „X‟ can be achived by using varignon‟s principle.

First, Take the moment @ A of all the forces.

$\mathbf{M}_{ALL}$= + (500 Sin 45° X 1.4) + (850 X 1.8) + (800 Sin 30° X 1.8) + 400

= + 3144.97 N-m [↴]______(1)

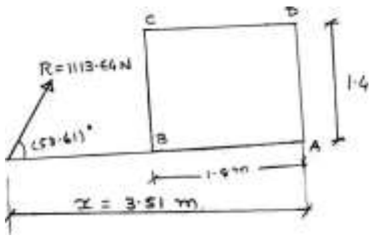
Now moment of „R‟ @ point „ A „

$\mathbf{M}_R$ = + (R Sin $\theta . X$) = + ($\sum Fy . x$) = $896.45 . x$ _______(2)

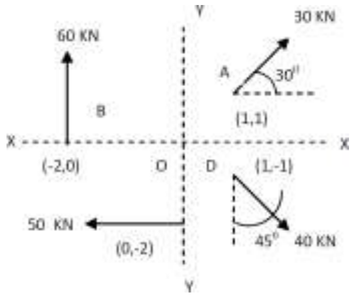
(1) = (2)

896.45 X = 3144.97

X = 3.51 m



Example: 9 Find magnitude, direction and location of resultant of force system with respect to point 'O' shown in fig.



Answer

Summation of horizontal forces

$$\Sigma H = +30\,Cos\,30^o - 50 + 40 Sin\,45^o = +4.265\,KN \quad (\rightarrow)$$

Summation of vertical forces

$$\Sigma V = +30\,Sin 30^o + 60 - 40\,Cos\,45^o = +46.72\,KN \quad (\uparrow)$$

Resultant force

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2} = \sqrt{(4.265)^2 + (46.72)^2} = 46.91\,KN$$

Angle of resultant

$$\tan\theta = \frac{46.72}{4.265}$$

$$\therefore \quad \theta = 84.78$$

Now,as we requred to find out the position of „R" with respect to the point „O". Take the moment of all the forces @ point „O „ we have,

M_0= +(30 Cos30° X 1) – (30 Sin30° X 1) + (60 X 2)+ (50 X 2)-(40 Cos45° X 1) + (40 Sin45° X 1)

M_0= + 230.98 KN- unit (↴)_________ (1)

Now, moment of „R" @ Pt. „O"

(considering „R" lies at a distance x from the point „O" in the horizontal direction)

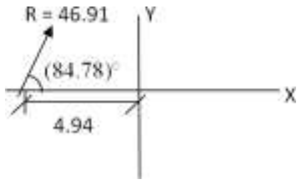
M_R = + (R Sinθ X) = (∑Fy.x)

M_R = +46.72.X ____________(2)

According to varignon's principle

$$\therefore 46.72\ X = 230.98$$

$$\therefore \ X = 4.94 \text{ unit}$$



Week - 4 & 5

Lecture

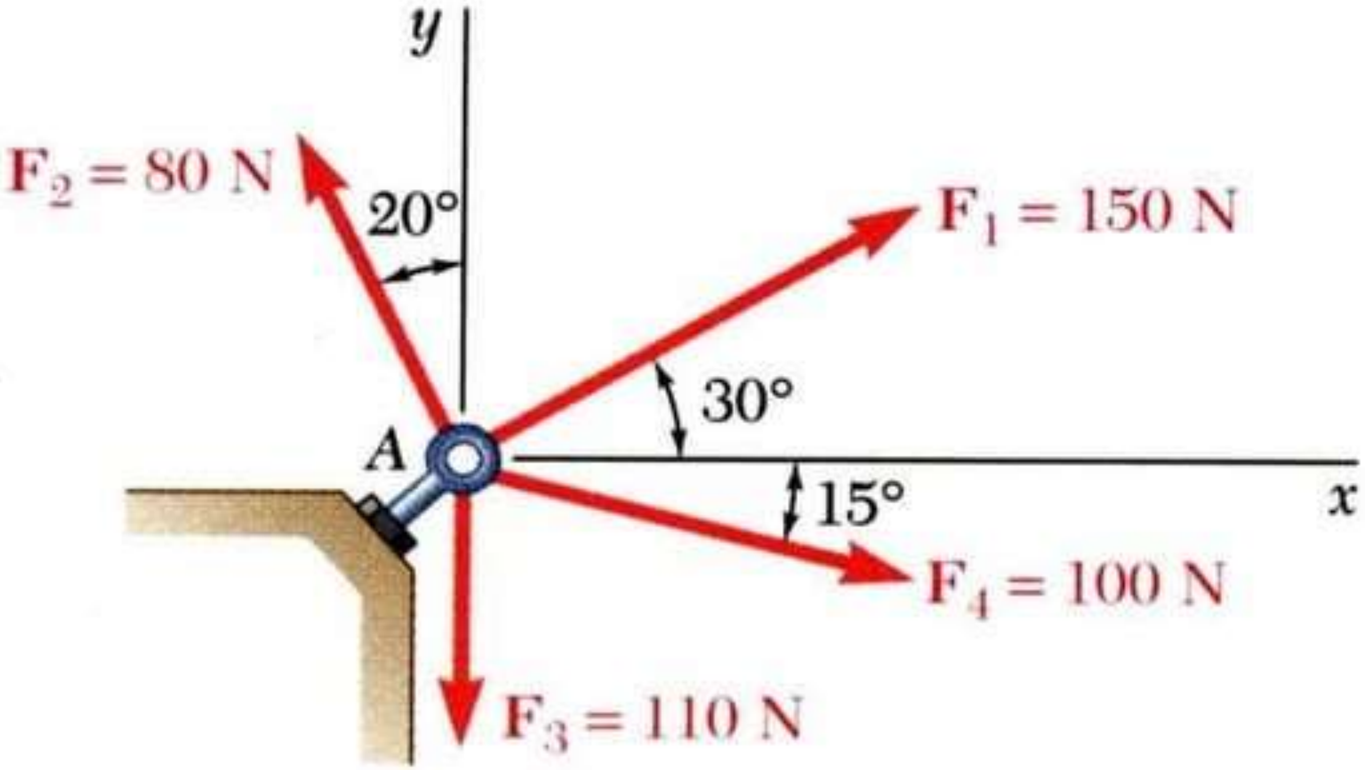
On

Review on Lami, Method of Resolution and Varignons Theorem

(64-75)

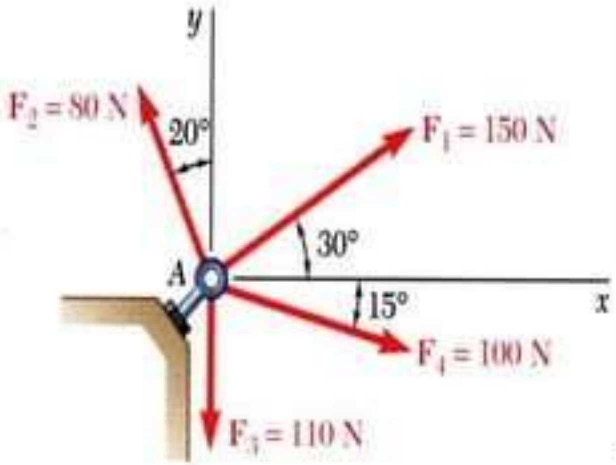
Problem

Determine the resultant of the following figure

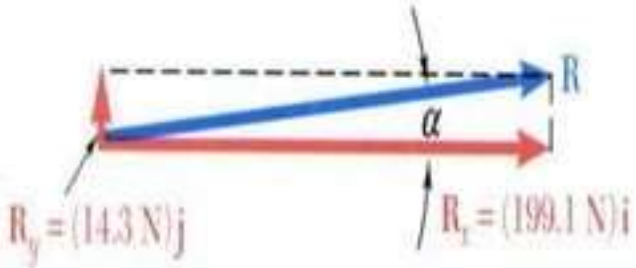


Problem – 2D- Concurrent - Resultant

Problem solution



Force	Mag	x − comp	y − comp
$\vec{F}_1$	150	$150\cos 30$	$+150 Sin 30$
$\vec{F}_2$	80	$-80\sin 20$	$+80 Cos 20$
$\vec{F}_3$	110	0	-110
$\vec{F}_4$	100	$+100\cos 15$	$-100\sin 15$
		$\sum F_x = +199.1$	$\sum F_y = +14.3$



Resultant is $R = \sqrt{\sum F_X^{\ 2} + \sum F_Y^{\ 2}}$

$$R = \sqrt{199.1^2 + 14.3^2}$$

$$R = 199.6\text{N}$$

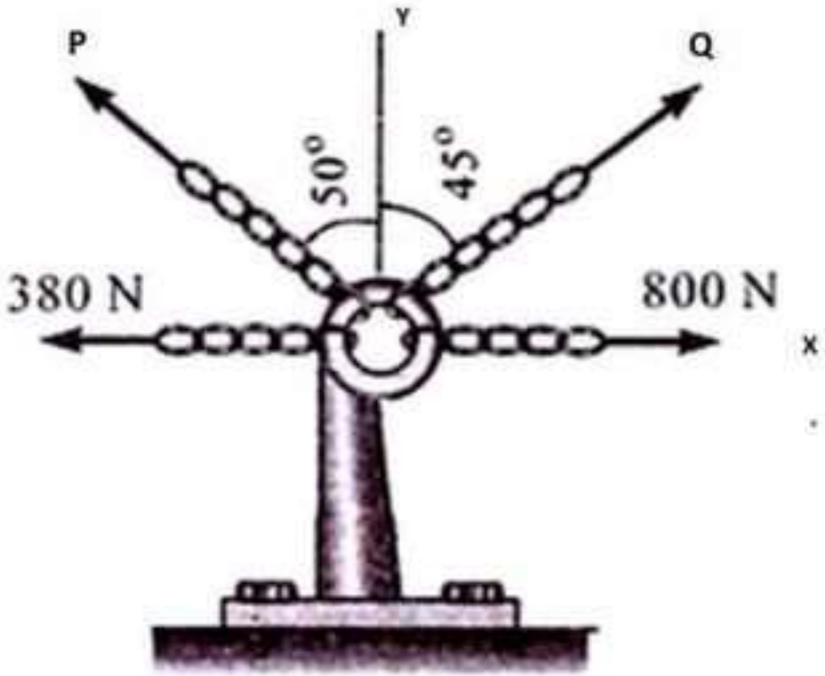
Direction is

$$\tan\alpha = \frac{14.3\,\text{N}}{199.1\,\text{N}}$$

$$\alpha = 4.1°$$

Problem

The resultant of the four concurrent forces as shown in Fig acts along Y-axis and is equal to 300N. Determine the forces P and Q.

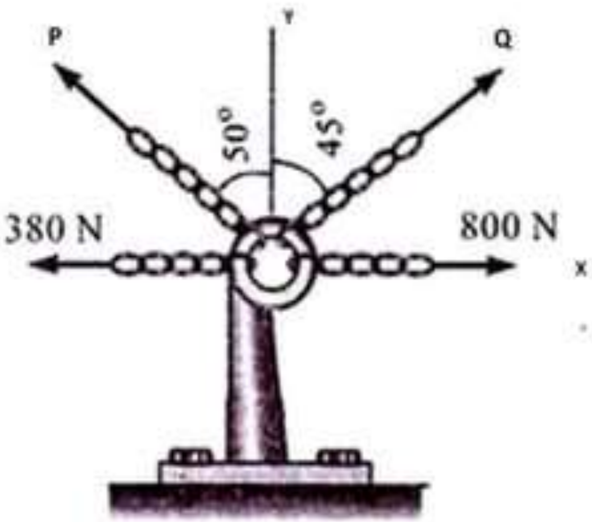


$$\sum F_x = 0$$

$$\sum F_y = R = 300N$$

Problem – 2D- Concurrent - Resultant

Problem solution



Force	Mag	x − comp	y − comp
$\vec{F}_1$	800	800	0
$\vec{F}_2$	380	−380	0
$\vec{F}_3$	Q	+ QSin45	+ QCos45
$\vec{F}_4$	P	− PSin50	+ PCos50

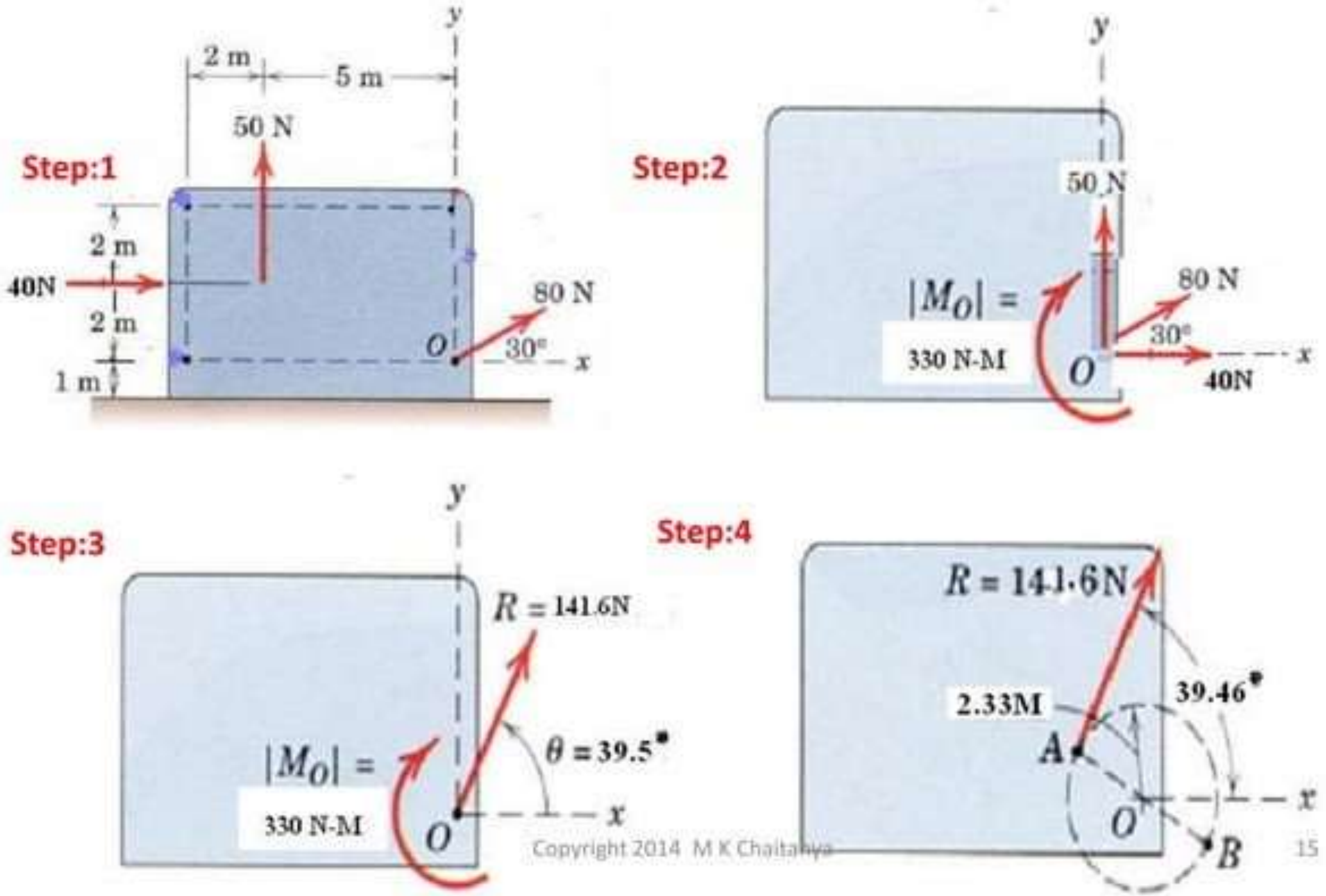
$$\sum F_x = 0 \qquad \sum F_x = 800 - 380 + QSin45 - P\sin 50 = 0$$

$$\sum F_y = R = 300N \qquad \sum F_y = QCos45 + PCos50 = R = 300$$

P = 511 N
Q =- 40.3N

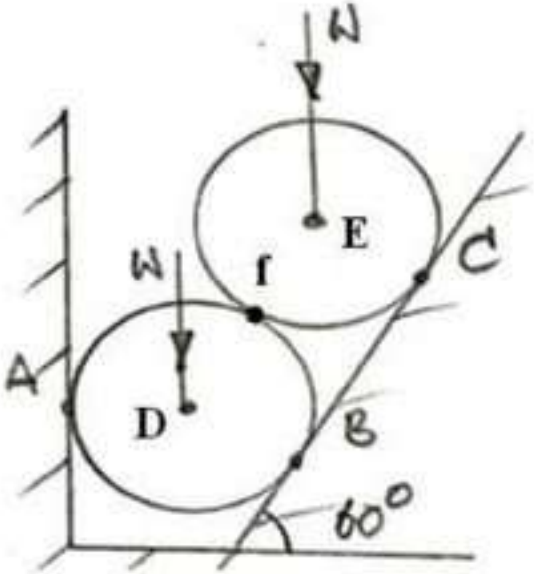
Problem solution **Resultant – Non-concurrent general forces in a plane**

Example: Determine the resultant force of the non-concurrent forces as shown in plate and distance of the resultant force from point 'O'.

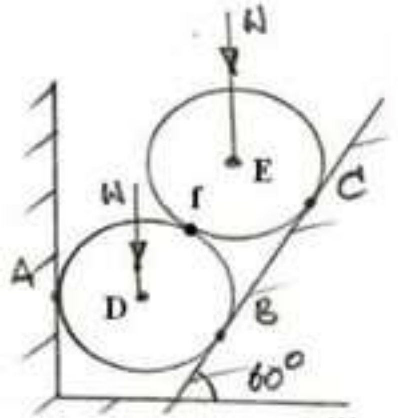


Problem

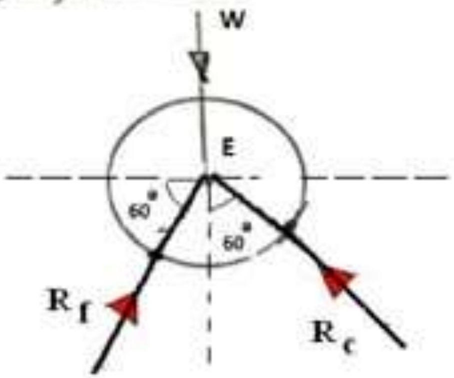
Find the reactions at A,B,C,D AND at F, Given W=100N



Problem – 2D - Concurrent - Equilibrium



W=100N
Find R_a, R_b and R_c

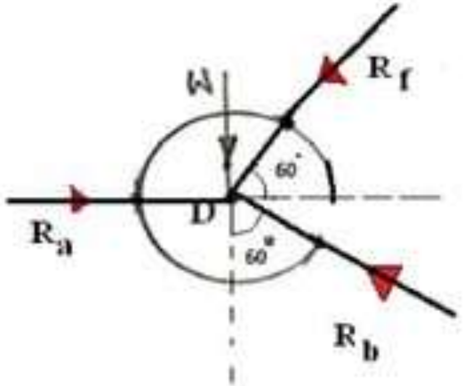


Since the body is in equilibrium and the forces are concurrent ……

$\sum F_X=0$; $R_f \cos 60^0 - R_c \sin 60^0 = 0$ ………1

$\sum F_Y=0$; $R_f \sin 60^0 + R_c \cos 60^0 - W(100) = 0$……2

R_f=86.6N
R_c=50N



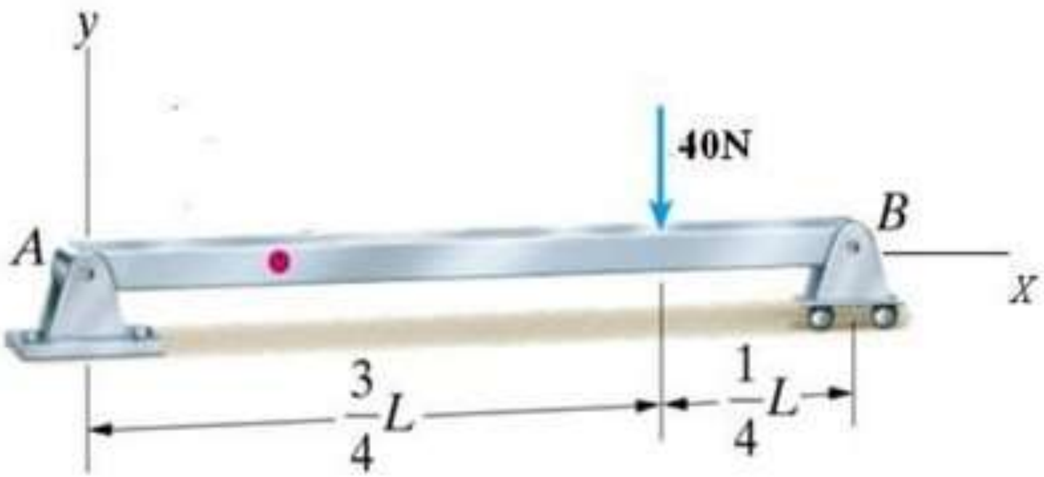
Since the body is in equilibrium and the forces are concurrent ……

$\sum F_X=0$; $-R_b \sin 60^0 - R_f \cos 60^0 + R_a = 0$ ………….1

$\sum F_Y=0$; $R_b \cos 60^0 - R_f \sin 60^0 - W = 0$………….2

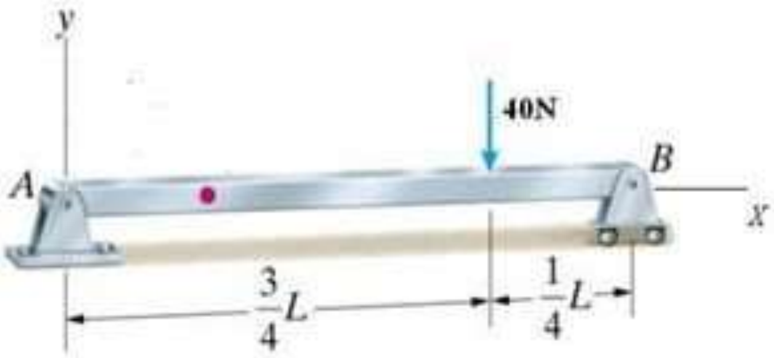
R_b = 350N
R_a = 346.4N

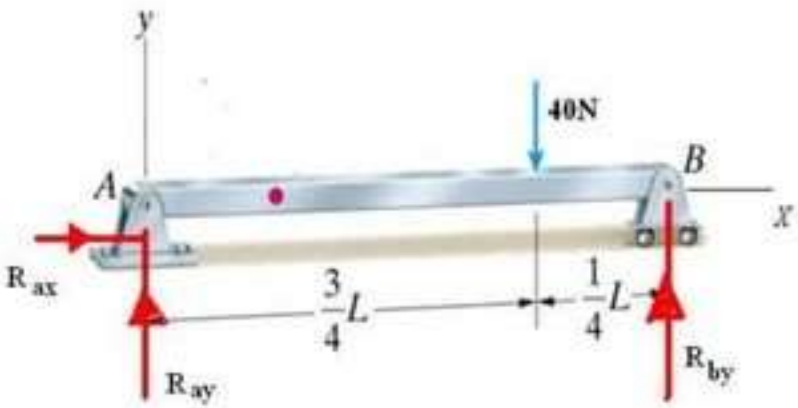
Problem Determine the reactions at A and B



Problem – 2D - Non Concurrent - Equilibrium

Problem solution





Since the body is in equilibrium and the forces are general forces then……

$\sum F_X=0$; $R_{ax} = 0$ ……………………………1

$\sum F_Y=0$; $R_{ay} + R_{by} - 40 = 0$……………2

$\sum M_A=0$; $(R_{by}*L) - 40*(3L/4) = 0$……3

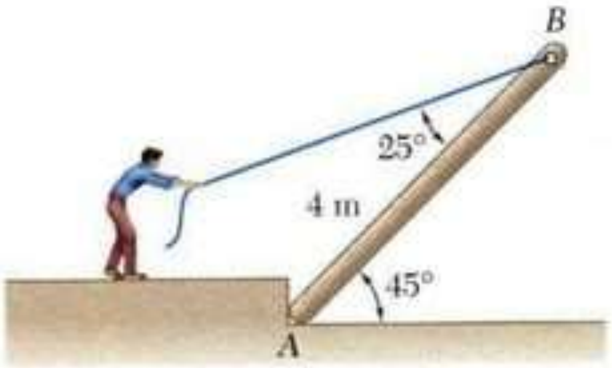
$R_{by}=30N$

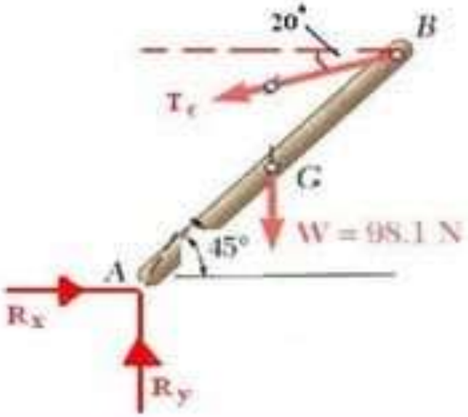
$R_{ay}=10N$

$R_{ax}=0N$

Problem

A man raises a 10 kg joist, of length 4 m, by pulling on a rope. Find the tension in the rope and the reaction at *A*.





Since the body is in equilibrium then.......

$\Sigma F_X=0$; $R_x - T_c*Cos20^0= 0$ 1

$\Sigma F_Y=0$; $R_y - T_c* Sin20^0 - W(98.1) = 0$............2

$\Sigma M_A=0$; $-(W*L/2) + (T_c*Cos20^0*4Sin45^0) -(Tc*Sin20^0*4*Cos45^0) = 0$......3

T_c=82N

R_x=77.1N

R_y=126.14N

Problem

Determine the reaction at **C** and the reaction at **E, Given P=200N**

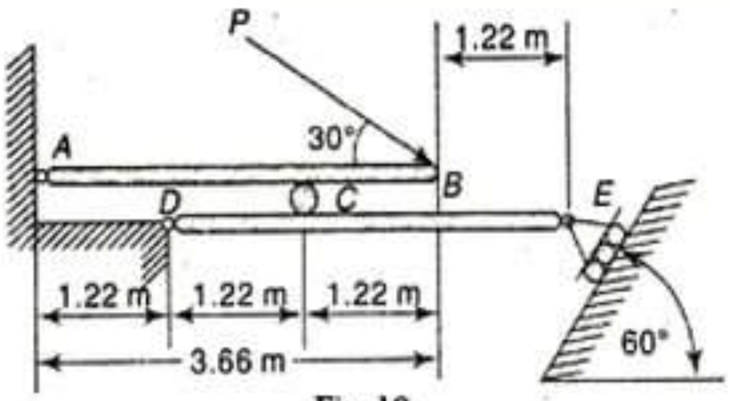
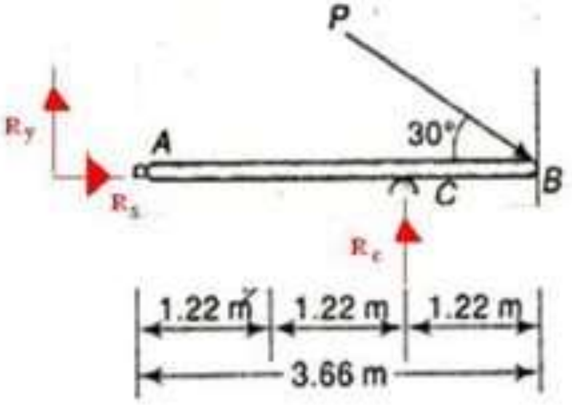


Fig. 10



From the freebody diagram of **AB** .Since the body is in equilibrium then...... ......

$\Sigma F_X=0$; $R_x + P*Cos30^0= 0$ 1

$\Sigma F_Y=0$; $R_y + R_c - PSin30^0 = 0$...............2

$\Sigma M_A=0$; $(R_c* 2.44) - (PSin30^{0*} 3.66) = 0$..........3

R_C=150N

CONTINUES........

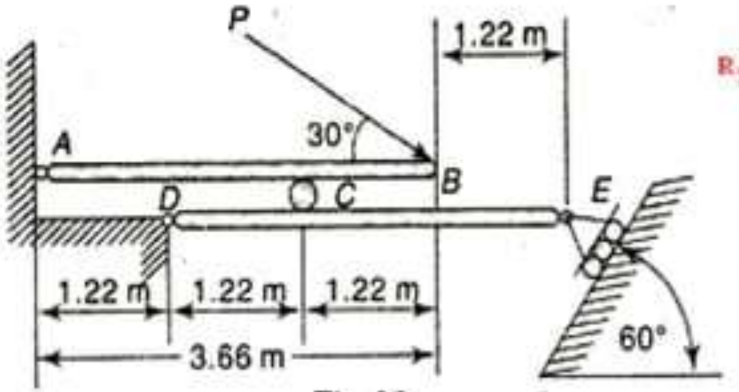
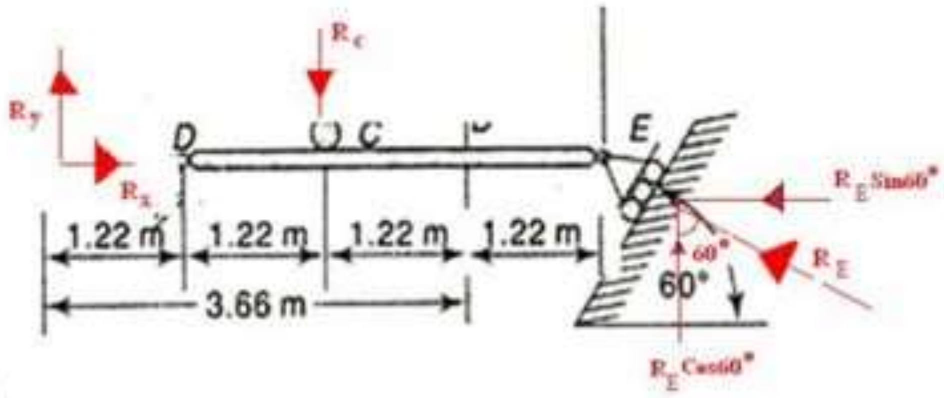


Fig. 10



From the freebody diagram of **DE** .Since the body is in equilibrium then…… ……

$\sum F_X=0$; $R_x - R_E \sin 60^0 = 0$ ………………1

$\sum F_Y=0$; $R_y - R_C + R_E \cos 60^0 = 0$……………2

$\sum M_D=0$; $(R_C * 1.22) - (R_E \cos 60^0 * 3.66) = 0$……….3

$R_E=100N$

Week -6

Lecture

On

Types of Supports in Structure

(77-84)

Types of Supports:-

1-Roller Supports:- can resist a vertical force but not a horizontal force. A roller support or connection is free to move horizontally as there is nothing constraining it.

Application: The most common use of a roller support is in a bridge. In civil engineering, a bridge will typically contain a roller support at one end to account for vertical displacement and expansion from changes in temperature. This is required to prevent the expansion causing damage to a pinned support.

Limitations: This type of support does not resist any horizontal forces. This obviously has limitations in itself as it means the structure will require another support to resist this type of force.

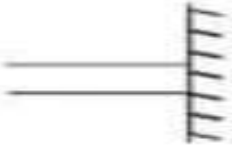
Example	Represented By	Reactions
	 ROLLER SUPPORT	Vertical Fixity: RFFRRR


HAER PA-93-11

2- Fixed Support:- A fixed support is the most rigid type of support or connection. It constrains the member in all translations and rotations (i.e. it cannot move or rotate in any direction). The easiest example of a fixed support would be a pole or column in concrete. The pole cannot twist, rotate or displace; it is basically restricted in all its movements at this connection.

Application: Fixed supports are extremely beneficial when you can only use a single support. The fixed support provides all the constraints necessary to ensure the structure is static. It is most widely used as the only support for a cantilever.

Limitations: Fixed supports offer absolutely no 'give'. In a sense, its greatest advantage can also be its downfall, as sometimes a structure requires a little deflection or 'play' to protect other surrounding materials. For instance, as concrete continues to gain its strength it also expands. So if a support is not designed correctly the expansion could lead to a reduction in durability.

Example	Represented By	Reactions
	 FIXED SUPPORT	Vertical, Horizontal, Moments Fixity: FFFFFF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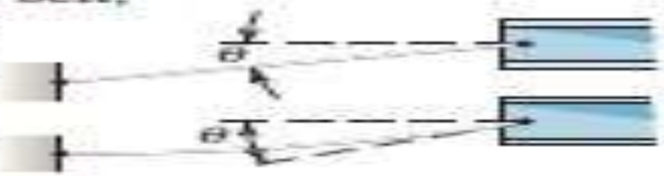
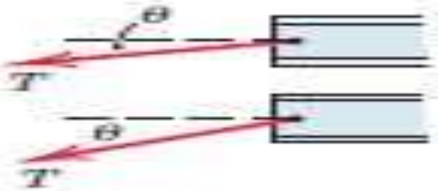
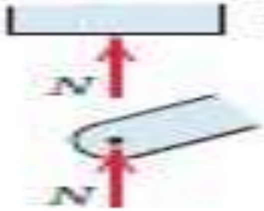
A pinned support is a very common type of support and is most commonly compared to a hingein civil engineering. Like a hinge, a pinned support allows rotation to occur but no translation (i.e. it resists horizontal and vertical forces but not a moment). Think of your elbow; you are able to extend and flex the elbow (rotation) but you cannot move your forearm left to right (translation).

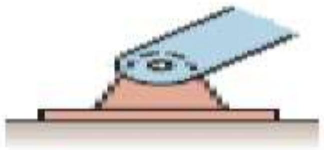
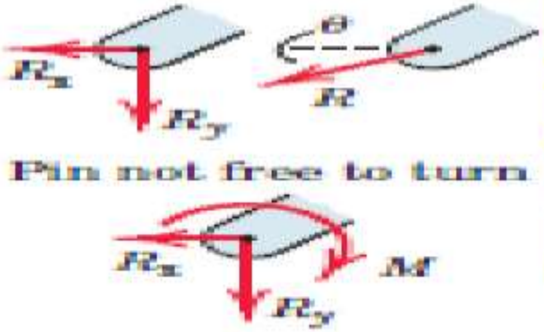
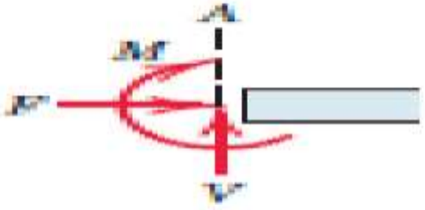
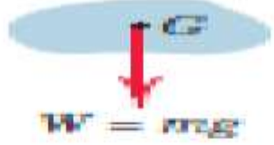
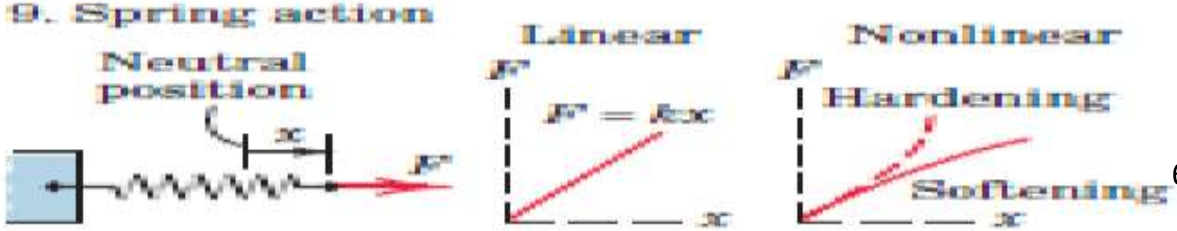
Application: Pinned supports can be used in trusses. By linking multiple members joined by hinge connections, the members will push against each other; inducing an axial force within the member. The benefit of this is that the members contain no internal moment forces, and can be designed according to their axial force only.

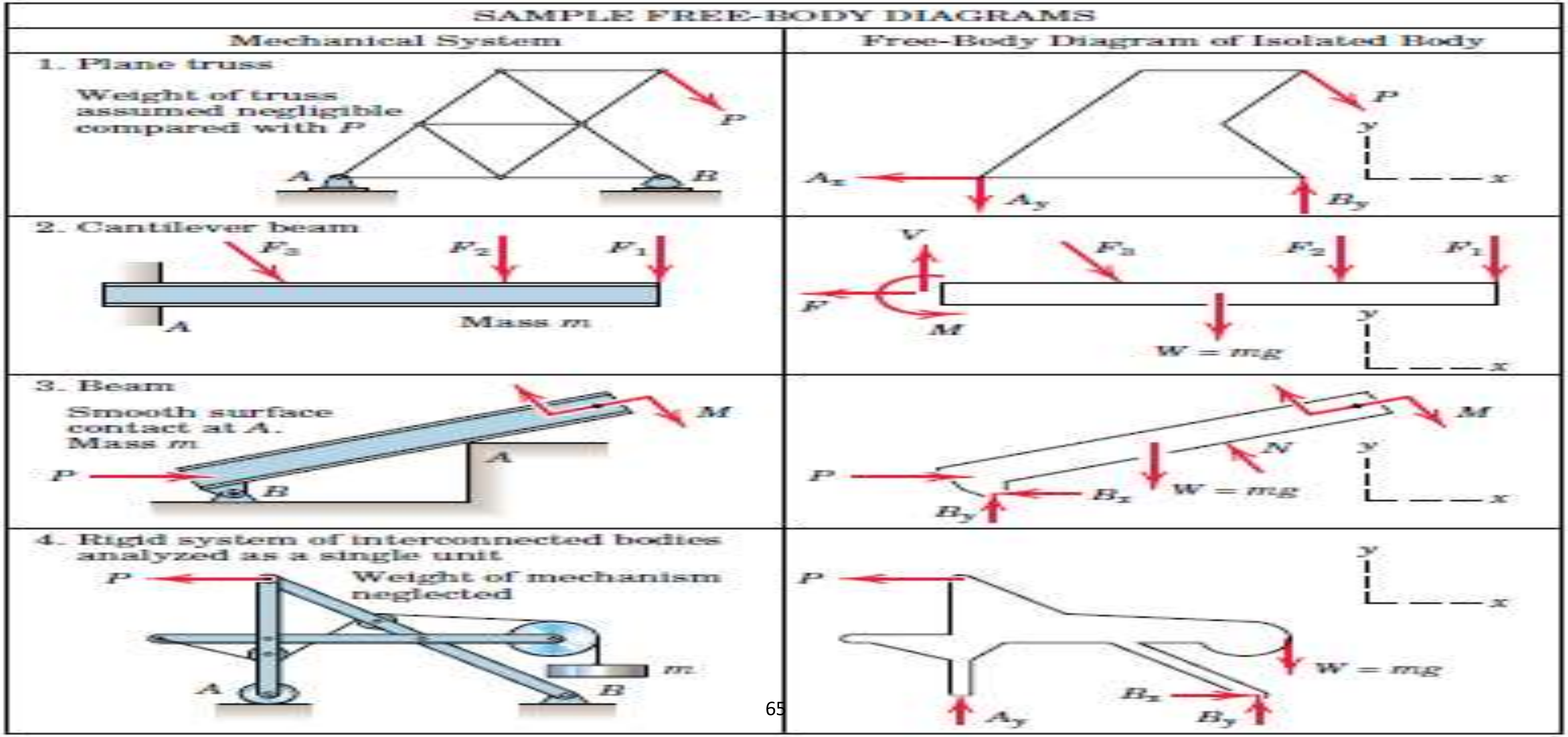
Limitations: A single pinned support cannot completely restrain a structure, as you need at least two supports to resist the moment.

Example	Represented By	Reactions
	 PIN SUPPORT	Vertical, Horizontal Fixity: FFFFRR

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential compo-nent *F* (frictional force) as well as a normal component *N* of the resultant
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (cont.)

Type of Contact and Force Origin	Action on Body to Be Isolated	
6. Pin connection 		A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 		The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 		Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.


SAMPLE FREE-BODY DIAGRAMS
Mechanical System
Free-Body Diagram of Isolated Body
1. Plane truss
Weight of truss assumed negligible compared with P
P
A
B
P
y
x
A_x
A_y
B_y
2. Cantilever beam
F_3
F_2
F_1
A
Mass m
V
F
M
F_3
F_2
F_1
W = mg
y
x
3. Beam
Smooth surface contact at A. Mass m
P
A
B
M
P
N
W = mg
B_x
B_y
M
y
x
4. Rigid system of interconnected bodies analyzed as a single unit
Weight of mechanism neglected
P
m
A
B
P
y
x
W = mg
B_x
B_y
A_y

Week -7

Lecture

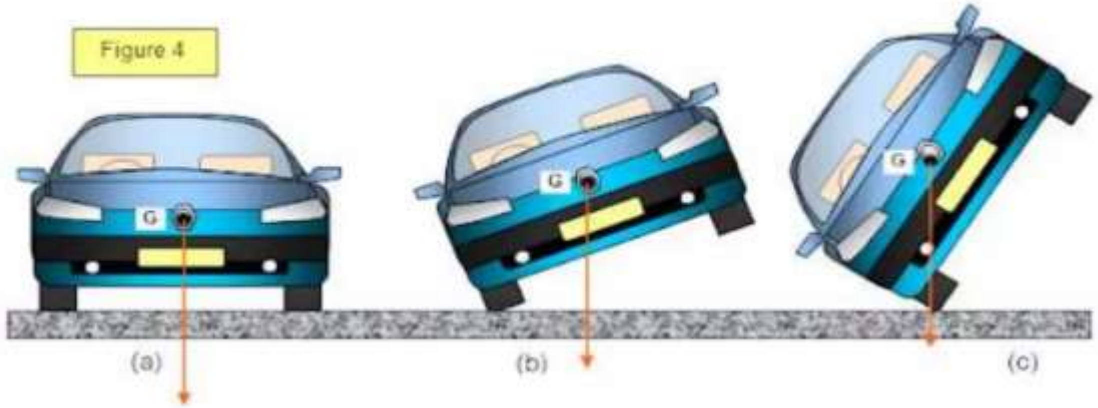
On

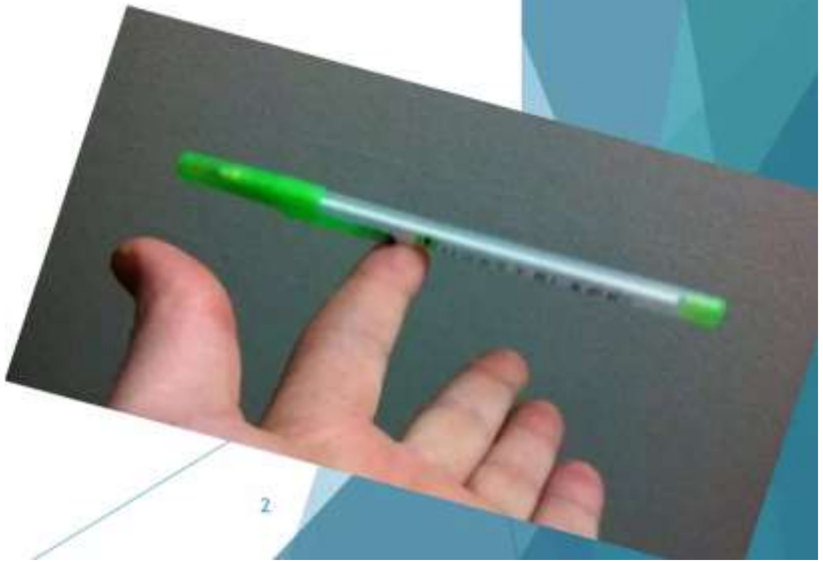
Centroid

(86-97)

CENTRE OF GRAVITY

- It is the point where the whole weight of the body is assumed to be concentrated. It is the point on which the body can be balanced.
- It is the point through which the weight of the body is assumed to act. This point is usually denoted by 'C.G.' or 'G'.

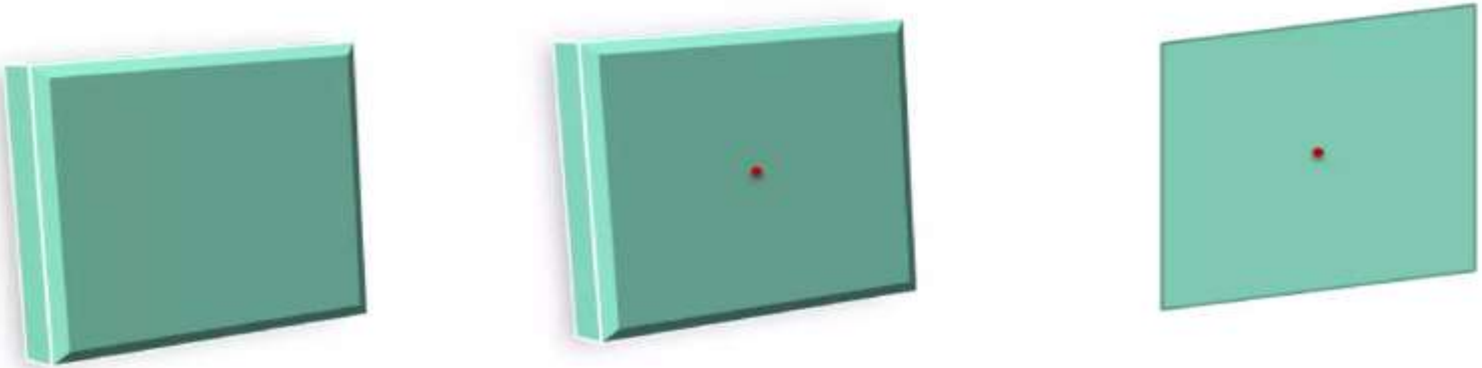




2

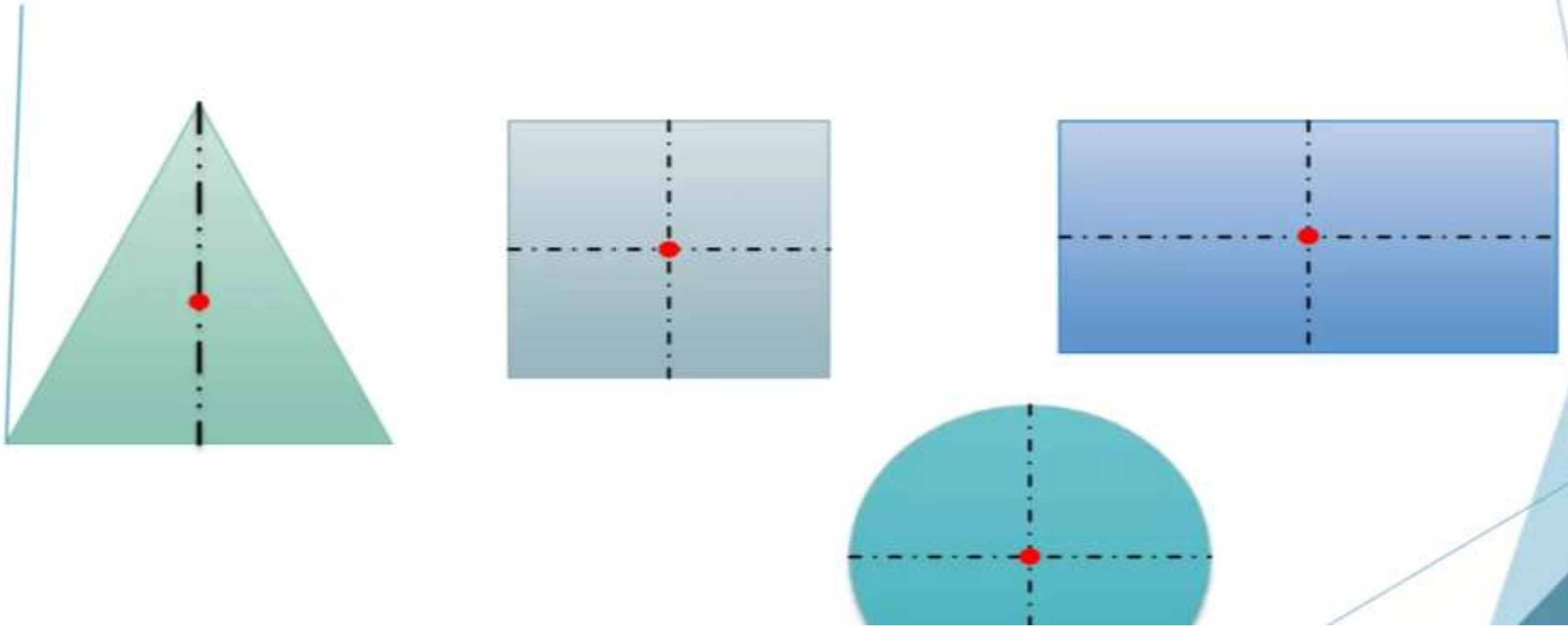
CENTROID

- Centroid is the point where the whole area of the plane figure is assumed to be concentrated.

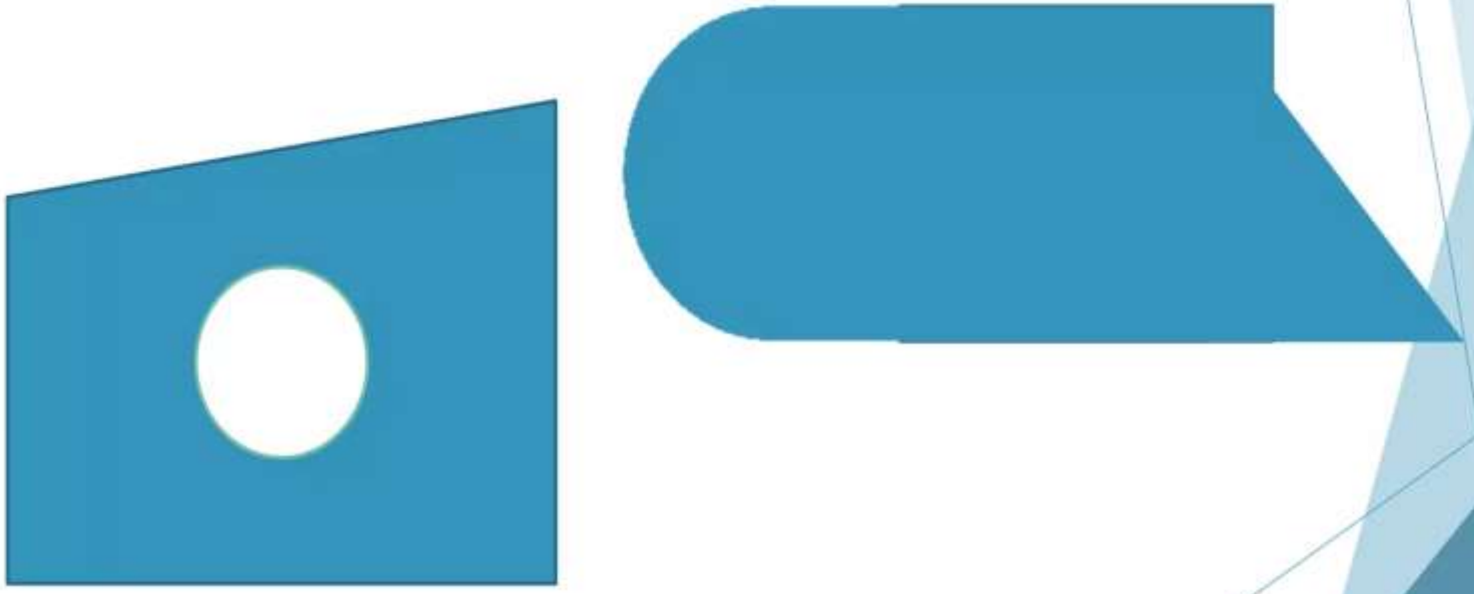


- It is easy to find the centroid of simple shapes.
- If the object has an axis of symmetry the centroid will always lie on that axis.
- If the object has two axes of symmetry, the centroid will be at the intersection of the two axes.

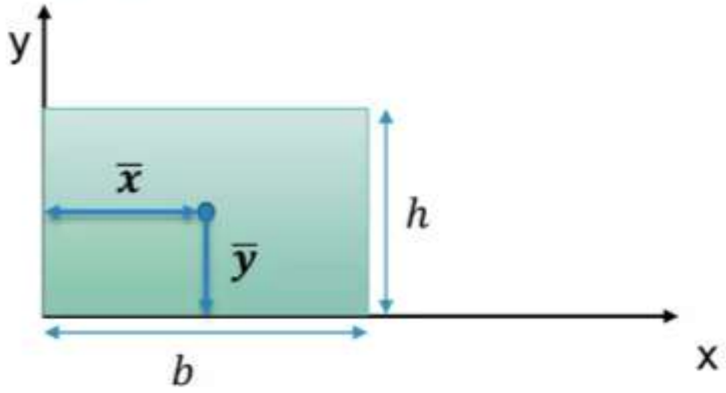
SIMPLE GEOMETRIC SHAPES



COMPOSITE GEOMETRIC SHAPES

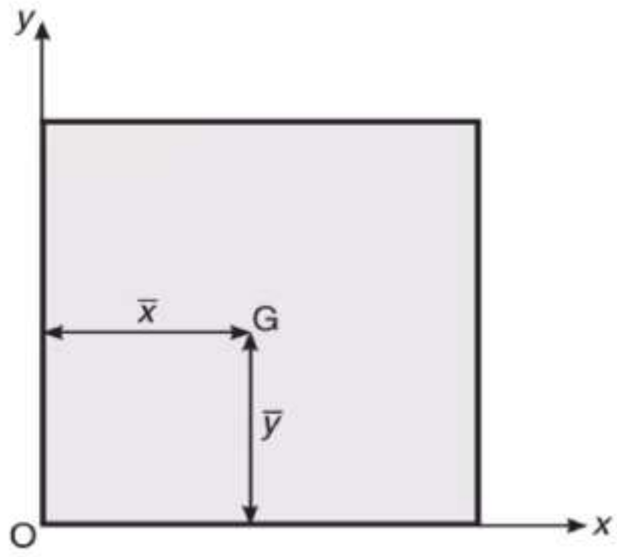


- CENTROID of a figure is always represented in a coordinate system as shown in figure below. The calculation of centroid means the determination of $\bar{x}$ and $\bar{y}$.



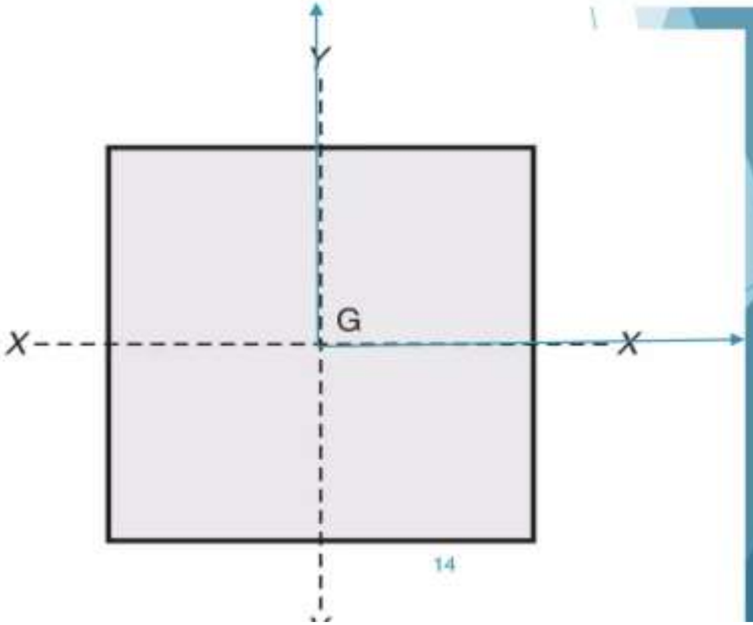
Axes of Reference

These are the axes with respect to which the centroid of a given figure is determined.

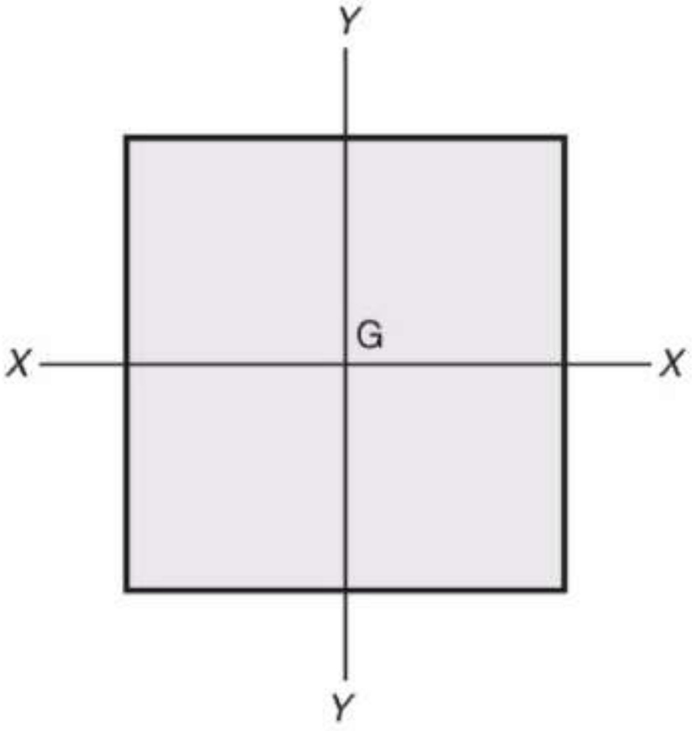


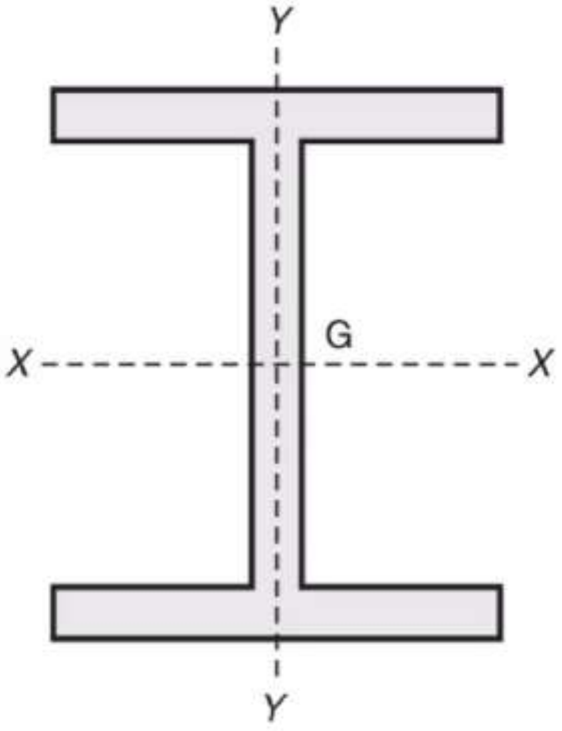
Centroidal Axis

The axis which passes through the centroid of the given figure is known as centroidal axis, such as
the axis X-X and the axis Y-Y shown in Figure.

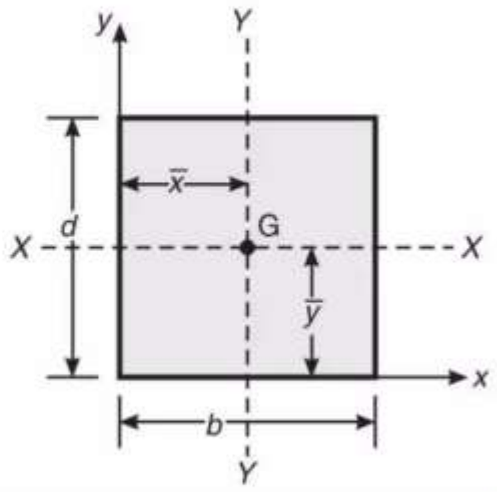


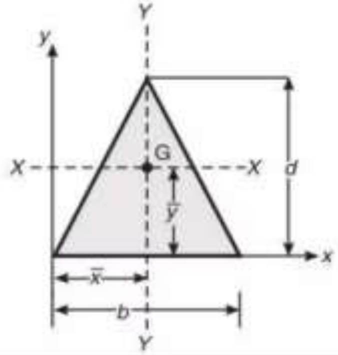
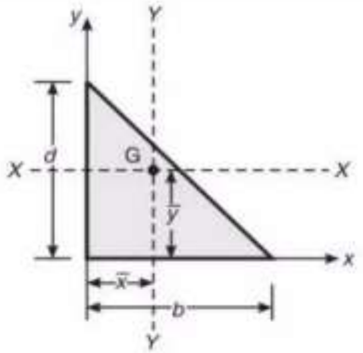
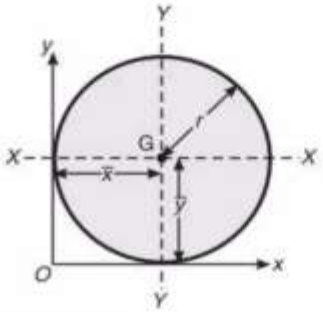
SYMMETRICAL AXES

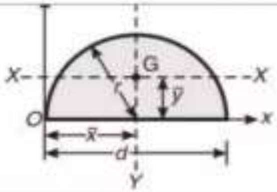
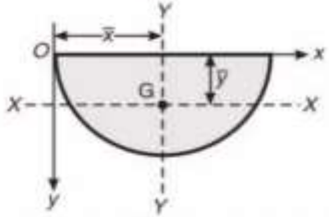
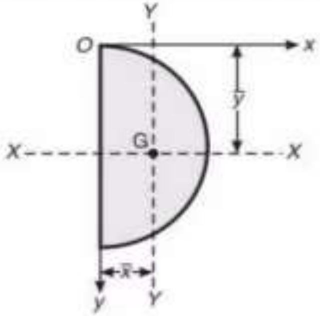
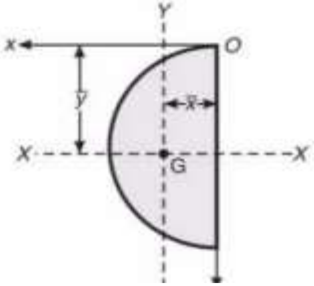


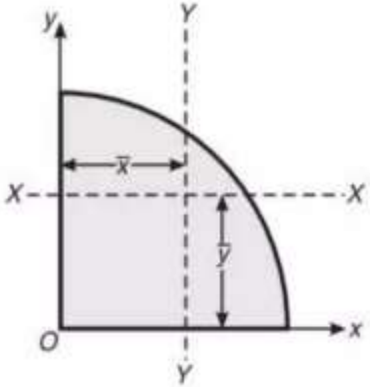


Centroids of Some Important Geometrical Figures

Shape	*Area*	$\overline{x}$	$\overline{y}$	*Figure*
Rectangle (Same for square)	bd	$\frac{b}{2}$	$\frac{d}{2}$	

Triangle	$\left(\frac{1}{2}\right)bd$	$\frac{b}{2}$	$\left(\frac{1}{3}\right)d$	
Right-angled triangle	$\left(\frac{1}{2}\right)bd$	$\left(\frac{1}{3}\right)b$	$\left(\frac{1}{3}\right)d$	
Circle	πr^2	$\overline{x} = r$	$\overline{y} = r$	

Semicircle	$\frac{\pi r^2}{2}$	$\frac{d}{2}$	$\frac{4r}{3\pi}$	
		$\frac{d}{2}$	$\frac{-4r}{3\pi}$	
		$\frac{4r}{3\pi}$	$\frac{-d}{2}$	
		$\frac{-4r}{3\pi}$	$\frac{d}{2}$	

Quarter circle	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	

Week -8

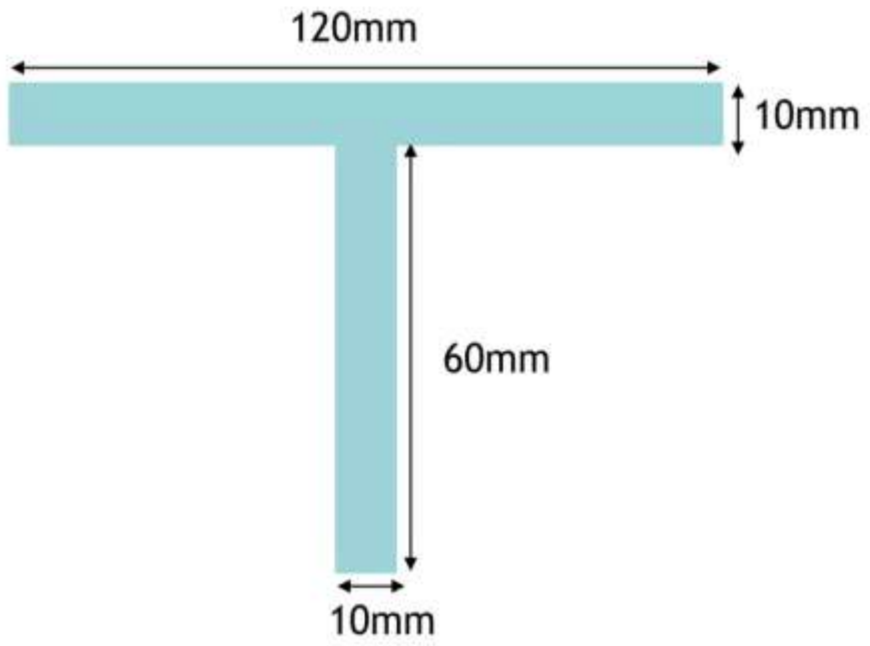
Lecture

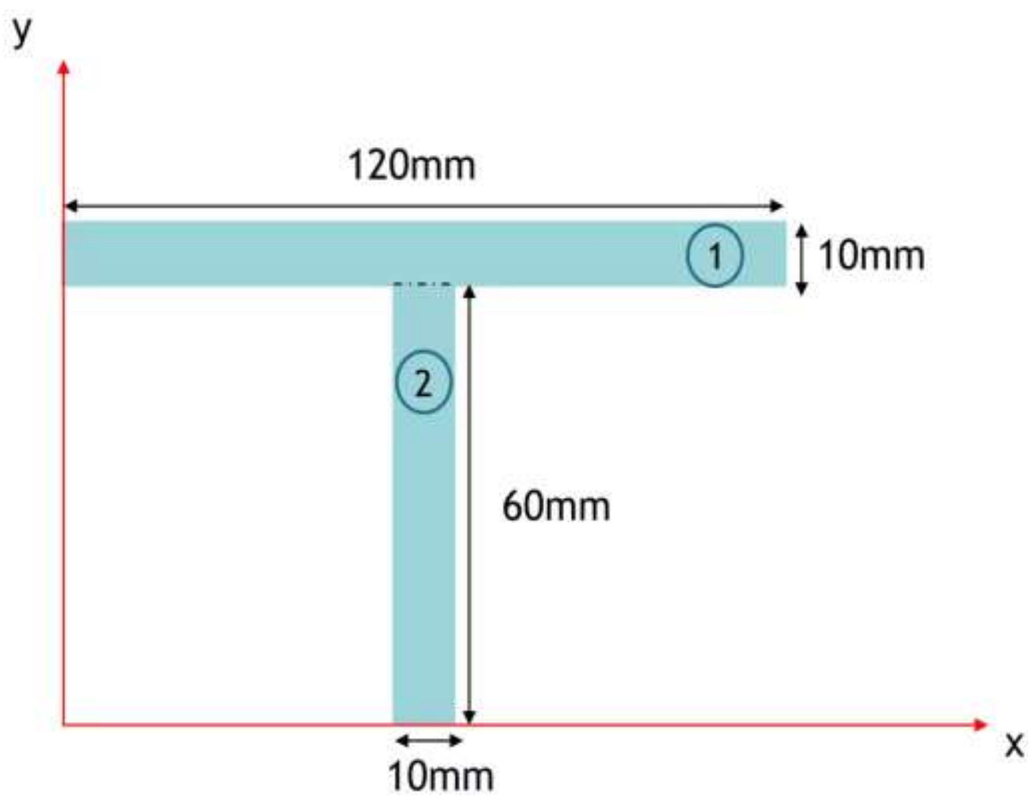
On

Centroid

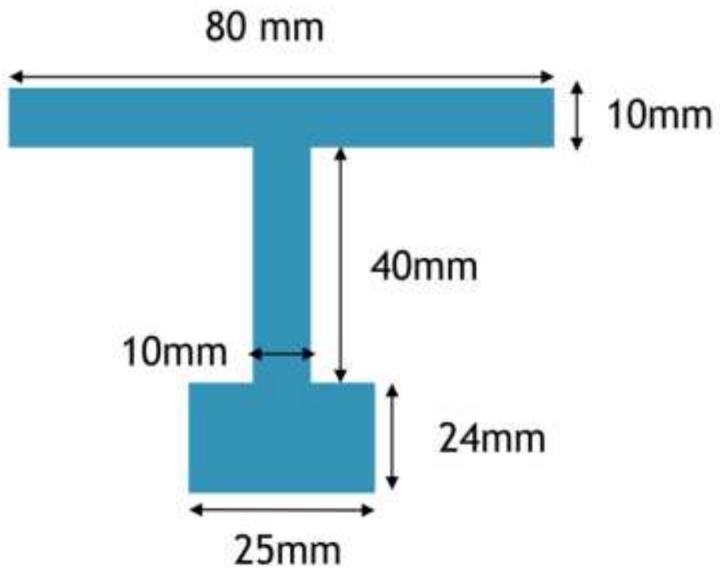
(99-106)

NUMERICAL 1

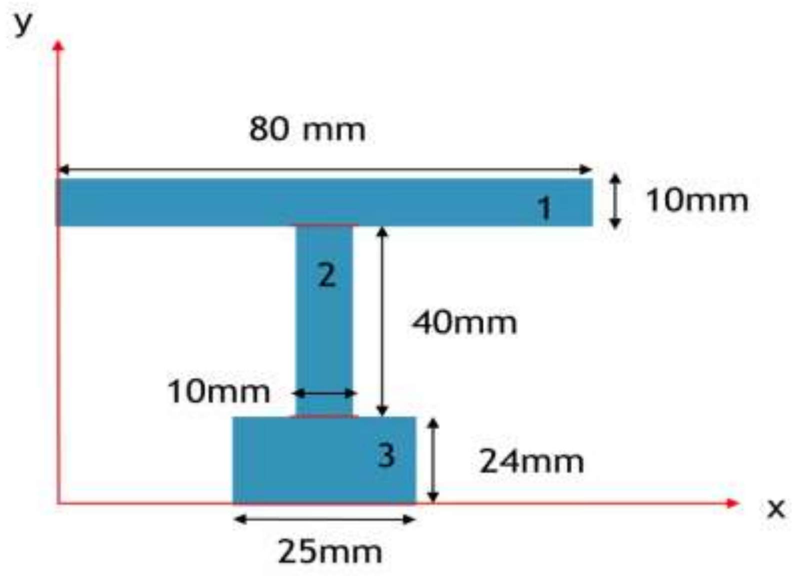



y
120mm
1
10mm
2
60mm
10mm
x

NUMERICAL 2



NUMERICAL 2

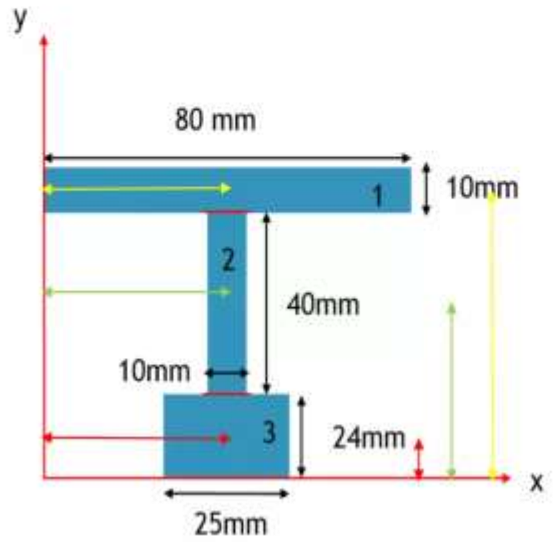


COMPONENTS	CENTROIDAL x DISTANCE (mm)	CENTROIDAL y DISTANCE (mm)	AREA (mm^2)	ax	ay
RECTANGLE 1	40	5+40 +24 = 69	80 x 10 = 800	32,000	55,200
RECTANGLE 2	40	20 +24 = 44	40 x 10 = 400	16,000	17,600
RECTANGLE 3	40	12	25 x 24 = 600	24,000	7,200

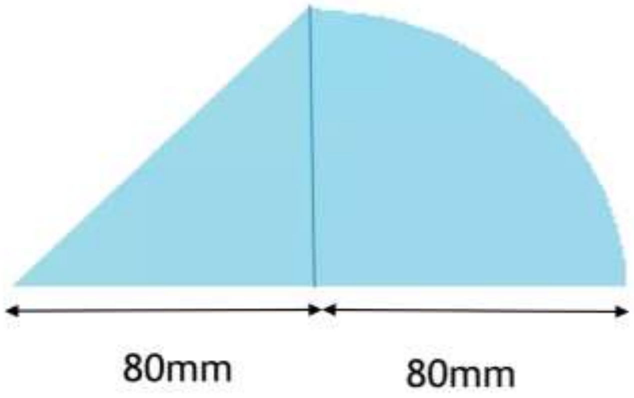
$\sum a = 1800$ $\sum ax = 72,000$ $\sum ay = 80,000$

$$\bar{x} = \frac{\sum ax}{\sum a} = \frac{72,000}{1800} = 40\text{mm}$$

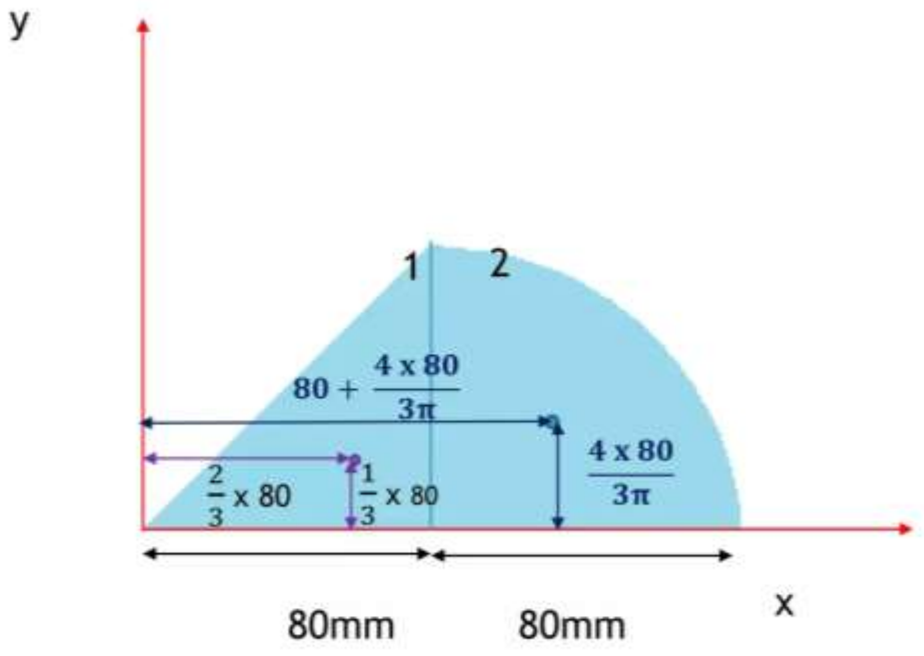
$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{80,000}{1800} = 44.44\text{mm}$$



NUMERICAL 5



NUMERICAL 5



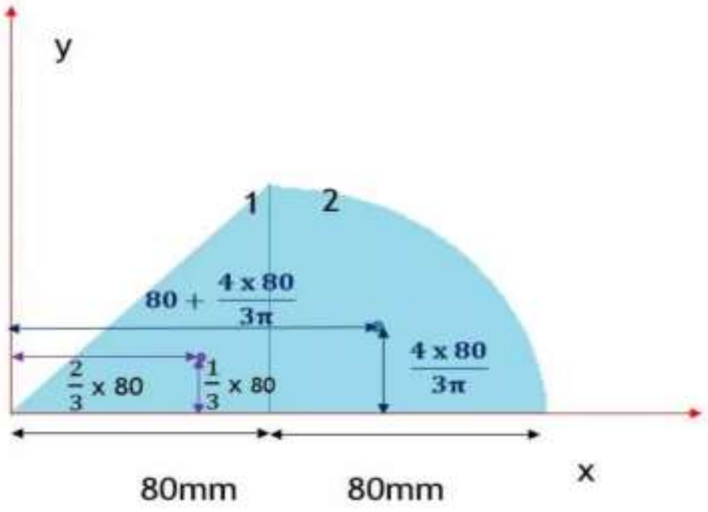
y
1
2
$80 + \frac{4 \times 80}{3\pi}$
$\frac{4 \times 80}{3\pi}$
$\frac{2}{3} \times 80$
$\frac{1}{3} \times 80$
80mm
80mm
x

COMPONENTS	CENTROIDAL x DISTANCE (mm)	CENTROIDAL y DISTANCE (mm)	AREA (mm^2)	ax	ay
TRIANGLE 1	$\frac{2}{3}$ x 80= 53.33	$\frac{1}{3}$ x 80 = 26.66	$\frac{1}{2}$ x 80 x 80 = 3200	1,70,656	85,312
QUARTER CIRCLE 2	$80 + \frac{4 \times 80}{3\pi}$ = 113.95	$\frac{4 \times 80}{3\pi}$ = 33.95	$\frac{\pi x 80^2}{4}$ = 5026.54	5,72,774.23	170651.03

$$\sum a = 8226.54 \quad \sum ax = 743430.23 \quad \sum ay = 2,55,963.03$$

$$\bar{x} = \frac{\sum ax}{\sum a} = \frac{743430.23}{8226.54} = \mathbf{90.37mm}$$

$$\bar{y} = \frac{\sum ay}{\sum a} = \frac{2,55,963.03}{8226.54} = \mathbf{31.11mm}$$



Week -9

Lecture

On

Centroid

(108-112)

CENTRE OF GRAVITY OF SYMMETRICAL SECTIONS

Section, whose centre of gravity is required to be found out, and is symmetrical about *X-X* axis or *Y-Y* axis the procedure for calculating the centre of gravity of the body is to calculate either $\bar{x}$ or $\bar{y}$. This is due to the reason that the centre of gravity of the body will lie on the axis of symmetry.

Example 4.1. Find the centre of gravity of a channel section 100 mm × 50 mm × 15 mm.

Solution: As the section is symmetrical about X-X axis, therefore its centre of gravity will lie on this axis. Now split up the whole section into three rectangles ABFJ, EGKJ and CDHK as shown in Fig 4.5. Let the face AC be the axis of reference.

(*i*) *Rectangle ABFJ*

$$a_1 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_1 = \frac{50}{2} = 25 \text{ mm}$$

(*ii*) *Rectangle EGKJ*

$$a_2 = (100 - 30) \times 15 = 1050 \text{ mm}^2$$

and $$x_2 = \frac{15}{2} = 7.5 \text{ mm}$$

(*iii*) *Rectangle CDHK*

$$a_3 = 50 \times 15 = 750 \text{ mm}^2$$

and $$x_3 = \frac{50}{2} = 25 \text{ mm}$$

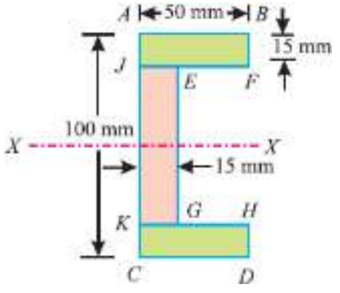


Fig 4.5

We know that distance between the centre of gravity of the section and left face of the section *AC*,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(750 \times 25) + (1050 \times 7.5) + (750 \times 25)}{750 + 1050 + 750} = 17.8 \text{ mm} \quad \textbf{Ans.}$$

Example 4.2 An I-section has the following dimensions in mm units:

Bottom flange = 300 × 100
Top flange = 150 × 50
Web = 300 × 50

Determine mathematically the position of centre of gravity of the section.

Solution. As the section is symmetrical about *Y-Y* axis, bisecting the web, therefore its centre of gravity will lie on this axis. Now split up the section into three rectangles as shown in Fig. 4.6

Let bottom of the bottom flange be the axis of reference.

(i) *Bottom flange*

$$a_1 = 300 \times 100 = 30\,000 \text{ mm}^2$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(ii) *Web*

$$a_2 = 300 \times 50 = 15\,000 \text{ mm}^2$$

and $$y_2 = 100 + \frac{300}{2} = 250 \text{ mm}$$

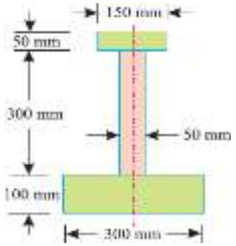


Fig 4.6

(iii) *Top flange*

$$a_3 = 150 \times 50 = 7500 \text{ mm}^2$$

and $$y_3 = 100 + 300 + \frac{50}{2} = 425 \text{ mm}$$

We know that distance between centre of gravity of the section and bottom of the flange,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(30\,000 \times 50) + (15\,000 \times 250) + (7500 \times 425)}{30\,000 + 15\,000 + 7500} = 160.7 \text{ mm} \qquad \textbf{Ans.}$$

Example 4.3 Find the centroid of the T-section as shown in figure 4.7 from the bottom

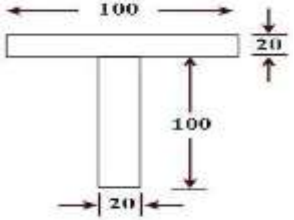


Fig 4.7

Soln:

Area (A_i)	x_i	y_i	A_i x_i	A_i y_i
2000	0	110	10,000	22,0000
2000	0	50	10,000	10,0000
4000			20,000	32,0000

$$y_c = \frac{\sum A_i y_i}{A_i} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{32,0000}{4000} = 80$$

Due to symmetry, the centroid lies on Y-axis and it is at distance of 80 mm from the bottom.

Example 4.3 Determine the centroid of the following figure.

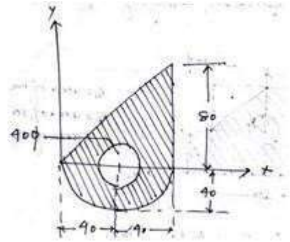


Fig 4.8

Soln:

$$A_1 = \text{Area of triangle} = \frac{1}{2} \times 80 \times 80 = 3200 m^2$$

$$A_2 = \text{Area of semicircle} = \frac{\pi d^2}{8} - \frac{\pi R^2}{2} = 2513.274 m^2$$

$$A_3 = \text{Area of semicircle} = \frac{\pi D^2}{2} = 1256.64 m^2$$

Area (A_i)	x_i	y_i	A_i x_i	A_i y_i
3200	$2\times(80/3)=53.33$	$80/3 = 26.67$	170656	85344
2513.274	40	$\frac{-4\times 40}{3\pi} = -16.97$	100530.96	-42650.259
1256.64	40	0	50265.6	0

$$x_c = \frac{A_1x_1 + A_2x_2 - A_3x_3}{A_1 + A_2 + A_3} = 49.57mm$$

$$y_c = \frac{A_1y_1 + A_2y_2 - A_3y_3}{A_1 + A_2 - A_3} = 9.58mm$$

Example 4.4 A body consists of a right circular solid cone of height 40 mm and radius 30 mm placed on a solid hemisphere of radius 30 mm of the same material. Find the position of centre of gravity of the body.

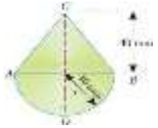


Fig 4.9

Solution. As the body is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis as shown in Fig. Let bottom of the hemisphere (D) be the point of reference.

Hemisphere

$$v_1 = \frac{2\pi}{3} \times r^3 = \frac{2\pi}{3}(30)^3 \text{ mm}^3$$

$$= 18\,000\,\pi \text{ mm}^3$$

$$y_1 = \frac{5r}{8} = \frac{5 \times 30}{8} = 18.75 \text{ mm}$$

Right circular cone

$$v_2 = \frac{\pi}{3} \times r^2 \times h = \frac{\pi}{3} \times (30)^2 \times 40 \text{ mm}^3$$

$$= 12\,000\,\pi \text{ mm}^3$$

$$y_2 = 30 + \frac{40}{4} = 40 \text{ mm}$$

Distance between centre of gravity of the body and bottom of hemisphere D,

$$\bar{y} = \frac{v_1\,y_1 + v_2\,y_2}{v_1 + v_2} = \frac{(18\,000\pi \times 18.75) + (12\,000\pi \times 40)}{18\,000\pi + 12\,000\pi} \text{ mm}$$

$= 27.3$ mm **Ans.**

CENTRE OF GRAVITY OF UNSYMMETRICAL SECTIONS

Sometimes, the given section, whose centre of gravity is required to be found out, is not symmetrical either about X-X axis or Y-Y axis. In such cases, we have to find out both the values of $\bar{x}$ and $\bar{y}$

Example 4.5. Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm. Solution :

As the section is not symmetrical about any axis, therefore we have to find out the values of *x* and *y* for the angle section. Split up the section into two rectangles as shown in Fig.4.10 Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

(i) *Rectangle* 1

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$x_1 = \frac{20}{2} = 10 \text{ mm}$$

and $$y_1 = \frac{100}{2} = 50 \text{ mm}$$

(ii) *Rectangle* 2

$$a_2 = (80 - 20) \times 20 = 1200 \text{ mm}^2$$

$$x_2 = 20 + \frac{60}{2} = 50 \text{ mm},$$

and $$y_2 = \frac{20}{2} = 10 \text{ mm}$$

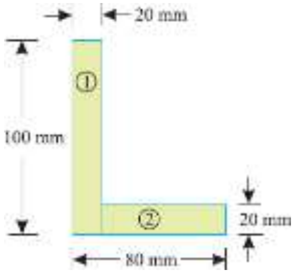


Fig. 4.10

We know that distance between centre of gravity of the section and left face,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200} = 25 \text{ mm} \quad \textbf{Ans.}$$

Similarly, distance between centre of gravity of the section and bottom face,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35 \text{ mm} \quad \textbf{Ans.}$$

Example 4.6. A semicircle of 90 mm radius is cut out from a trapezium as shown in Fig 4.11 .Find the position of the centre of gravity of the figure.

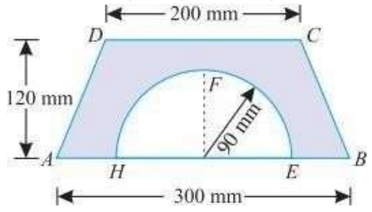


Fig 4.11

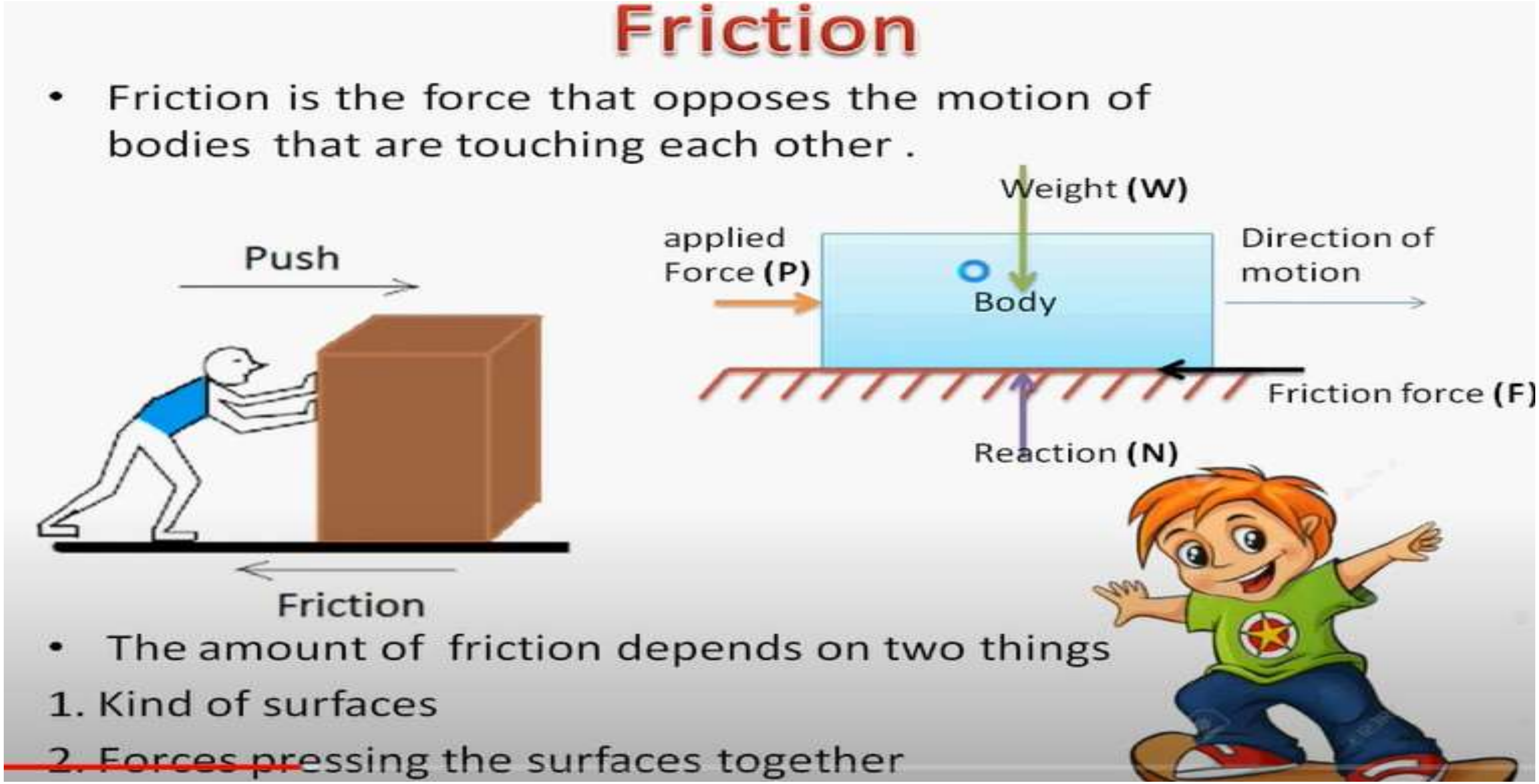
Week – 10 & 11

Lecture

On

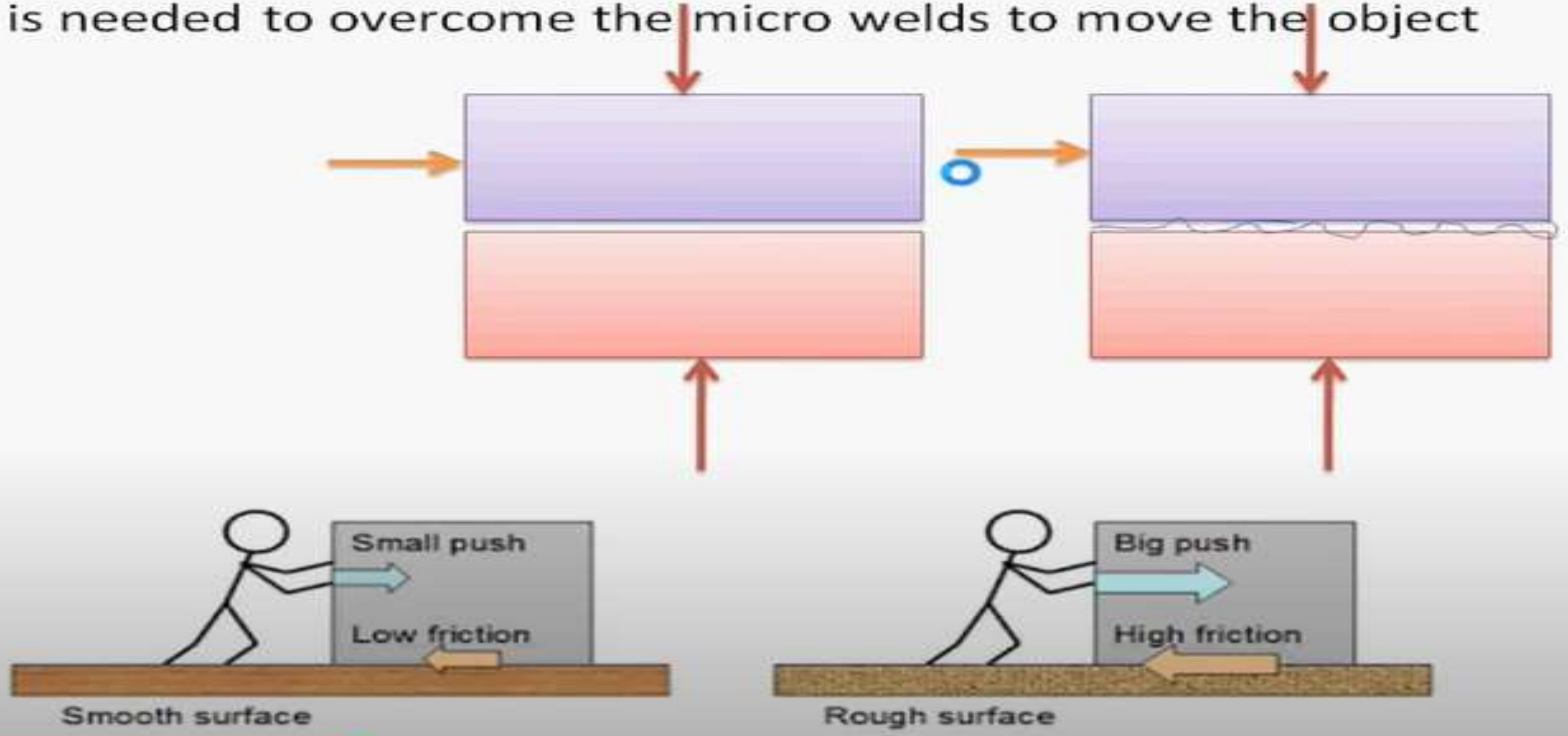
Friction

(113-137)


Friction
• Friction is the force that opposes the motion of bodies that are touching each other .
Push
Friction
Weight (W)
applied Force (P)
Body
Direction of motion
Friction force (F)
Reaction (N)
• The amount of friction depends on two things
1. Kind of surfaces
2. Forces pressing the surfaces together

Causes Of Friction

- Greater the force on object greater the force of welds and greater force is needed to overcome the micro welds to move the object



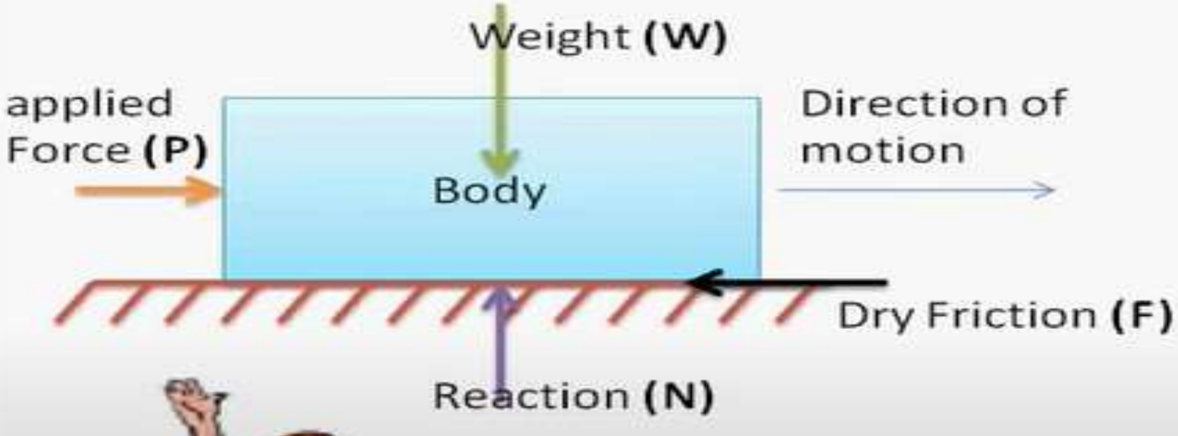
Types of Friction:

- There are several types of friction:

1. Dry friction
2. Fluid friction
3. Lubricated friction
4. Statics friction
5. Dynamic friction
6. Sliding friction
7. Rolling friction
8. Limiting friction

Dry Friction:

- Friction between dry surfaces in contact is called dry friction.
- It is also called **coulomb friction.**

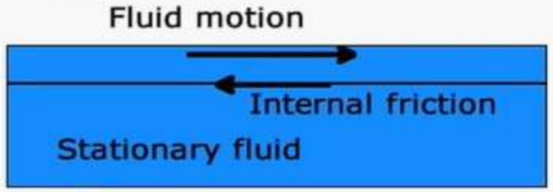


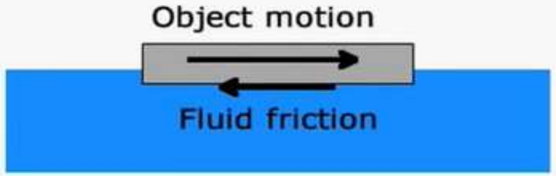
Fluid Friction :

- **Fluid friction** is the force that resists motion either within the **fluid** itself or of another medium moving through the **fluid**.

- There is **internal friction**, which is a result of the interactions between molecules of the **fluid**, and

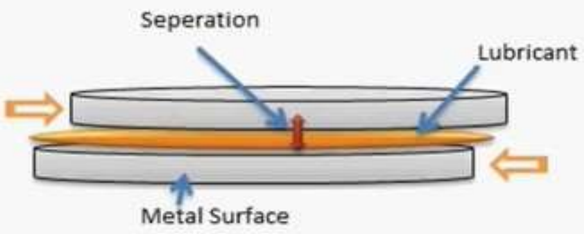
- there is **external friction**, which refers to how a **fluid** interacts with other matter.





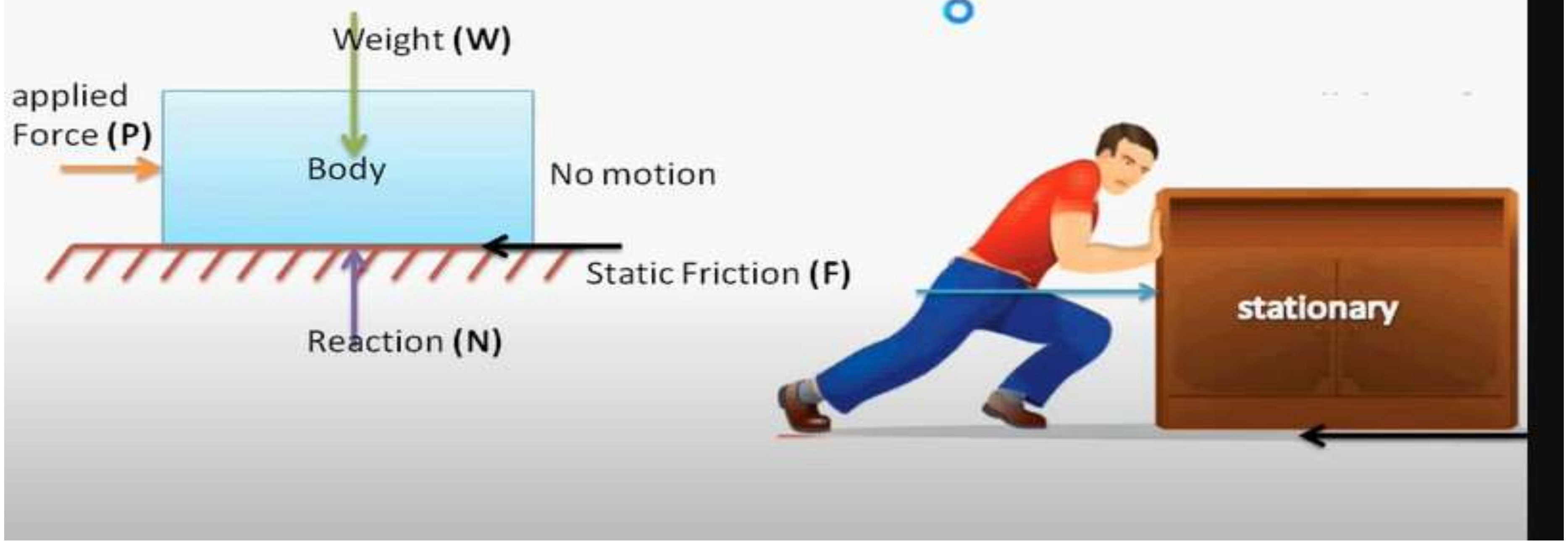
lubricated friction:

•**Lubricated friction** is a case of fluid **friction** where a **lubricant** fluid separates two solid surfaces.



Static friction:

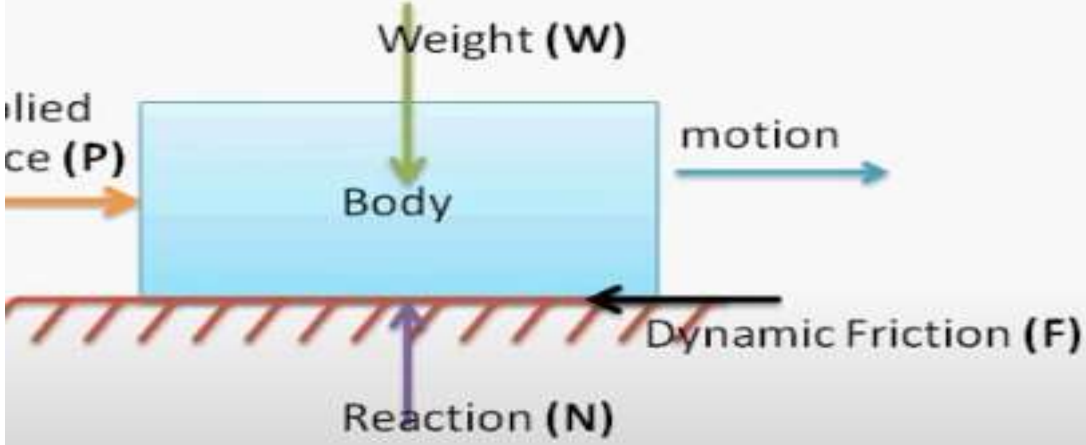
- The **friction** experienced when individuals try to move a stationary object on a surface, without actually triggering any relative motion between the body and the surface which it is on.

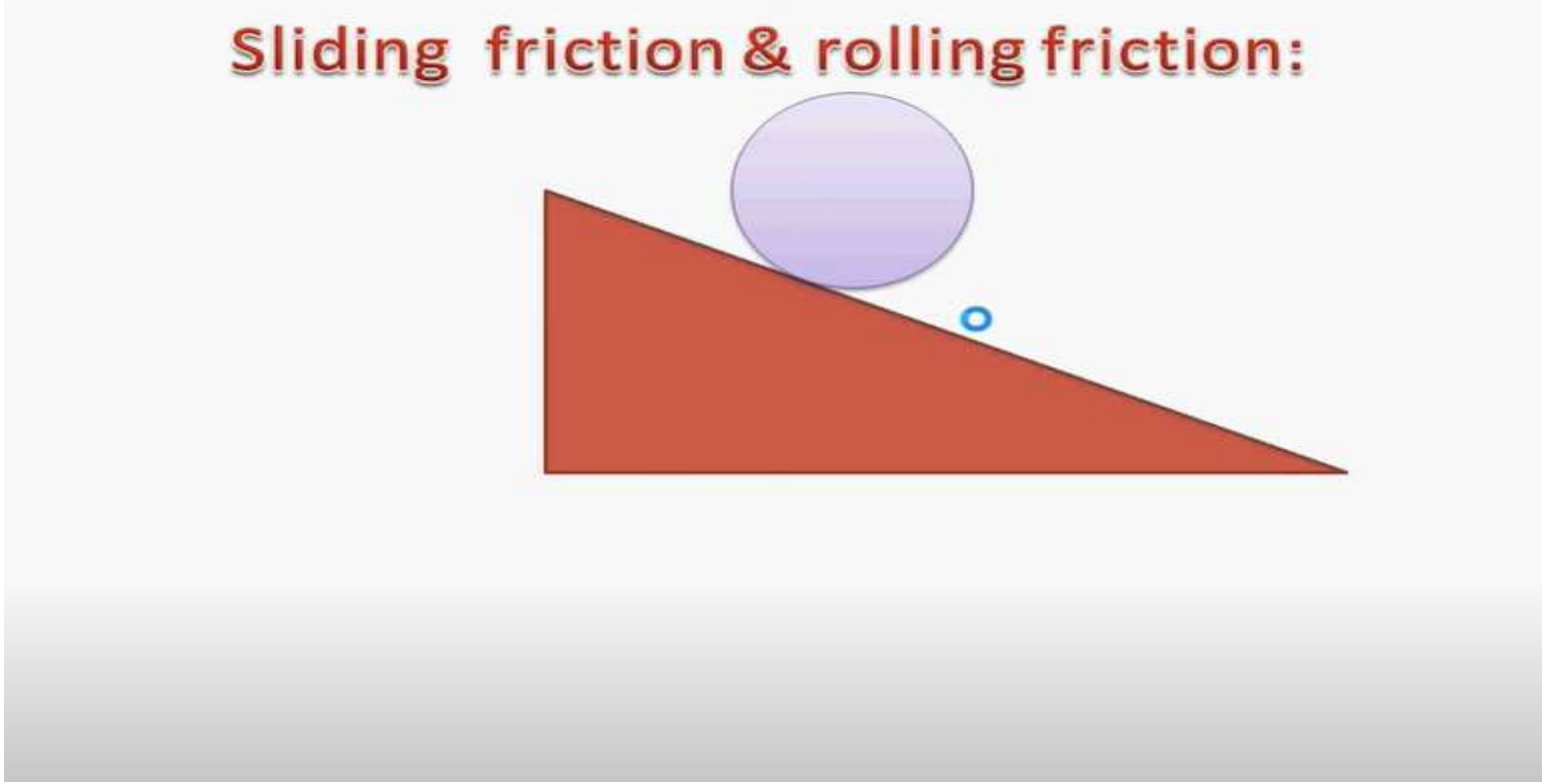


Dynamic friction:

- **dynamic friction** , occurs when two objects are moving relative to each other and rub together (like a sled on the ground).

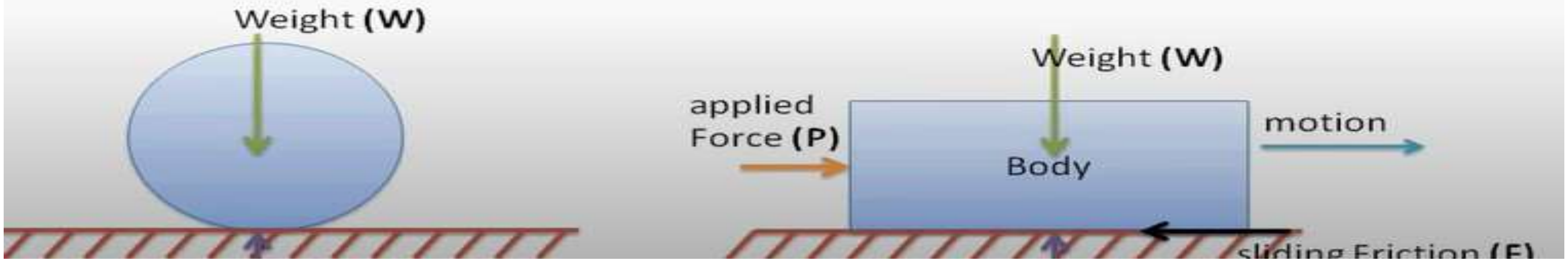
- also known as **Kinetic friction,**




Sliding friction & rolling friction:

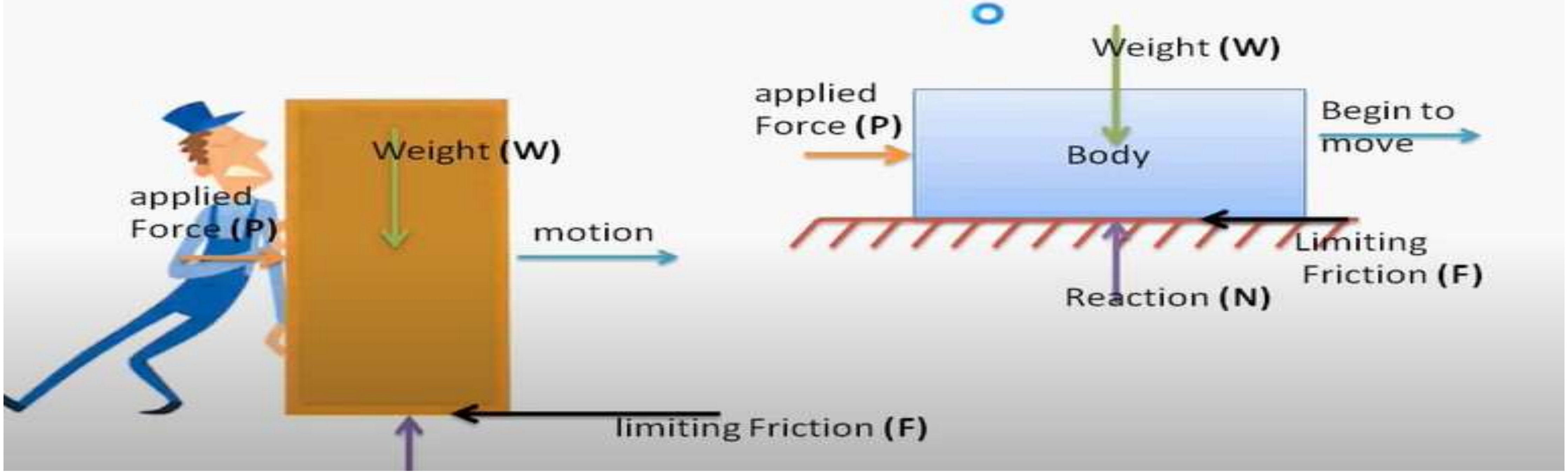
Sliding friction & rolling friction:

- Sliding Friction:-
  - Friction experienced by a body when it slides over another body, is called sliding friction.
- Rolling Friction:-
  - Friction experienced by a body when it rolls over another body is called rolling friction.



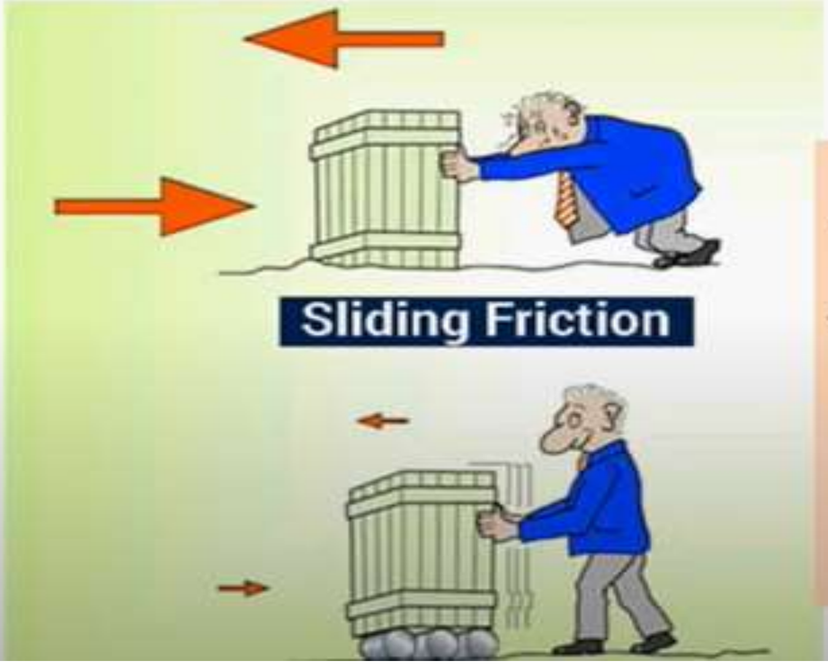
Limiting friction :

- The maximum friction force that can be developed at the contact surface, when body is just on the point of moving Is called limiting force of friction .



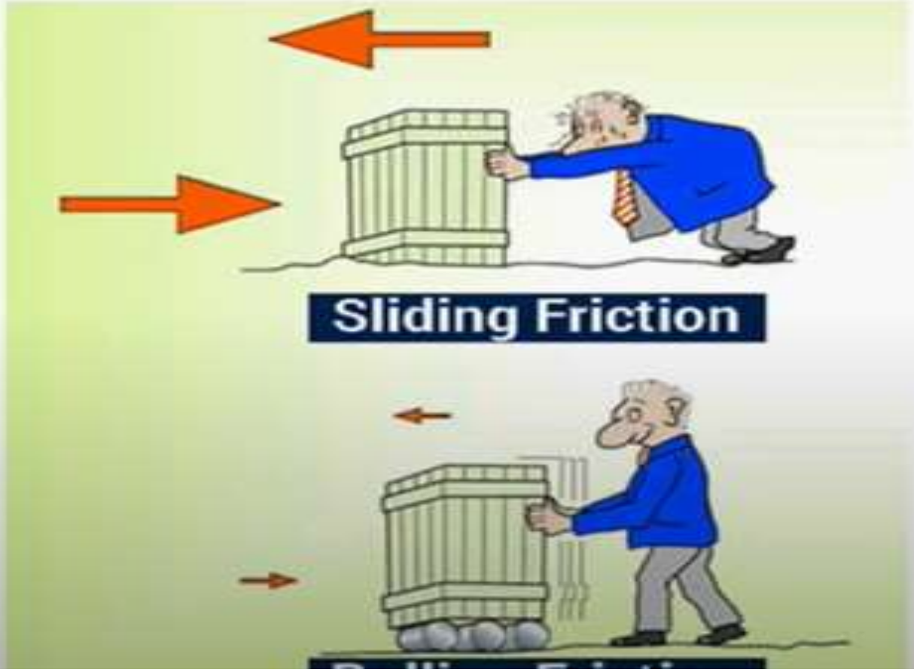
Static & Dynamic friction :

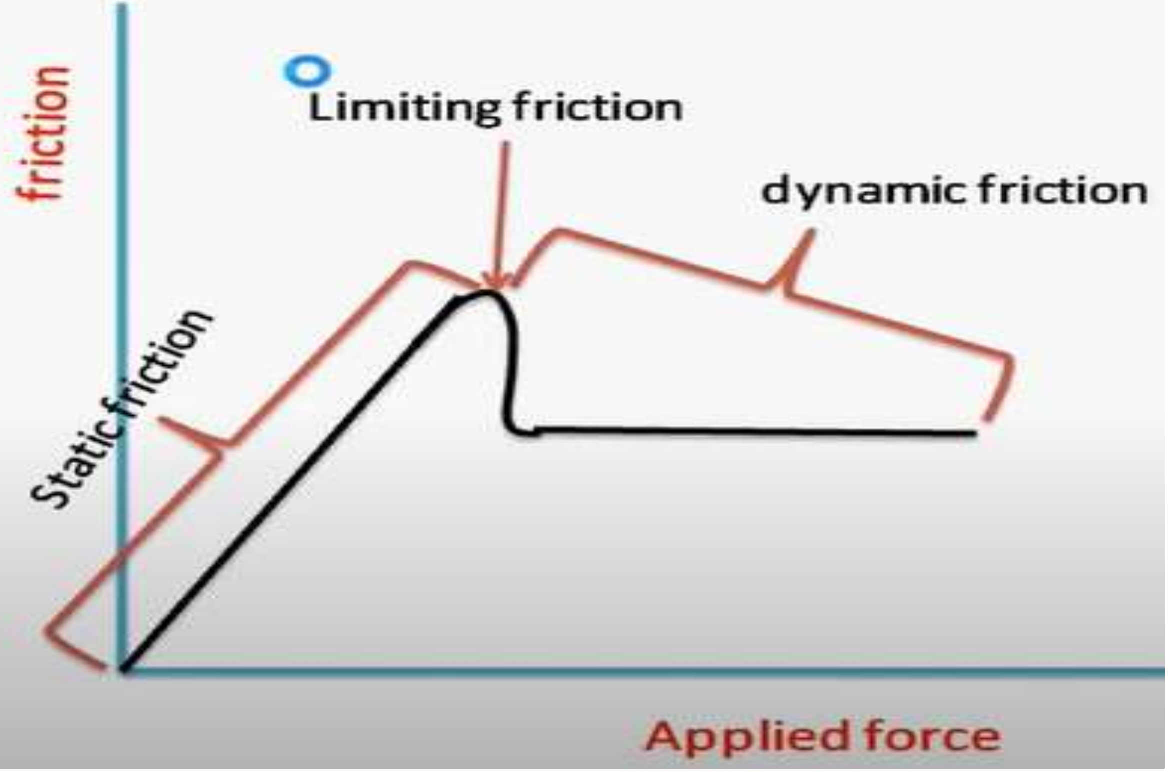
Sliding friction > rolling friction



Static & Dynamic friction :

Sliding friction	> rolling friction
limiting friction	> dynamic friction
Dry friction	> fluid friction

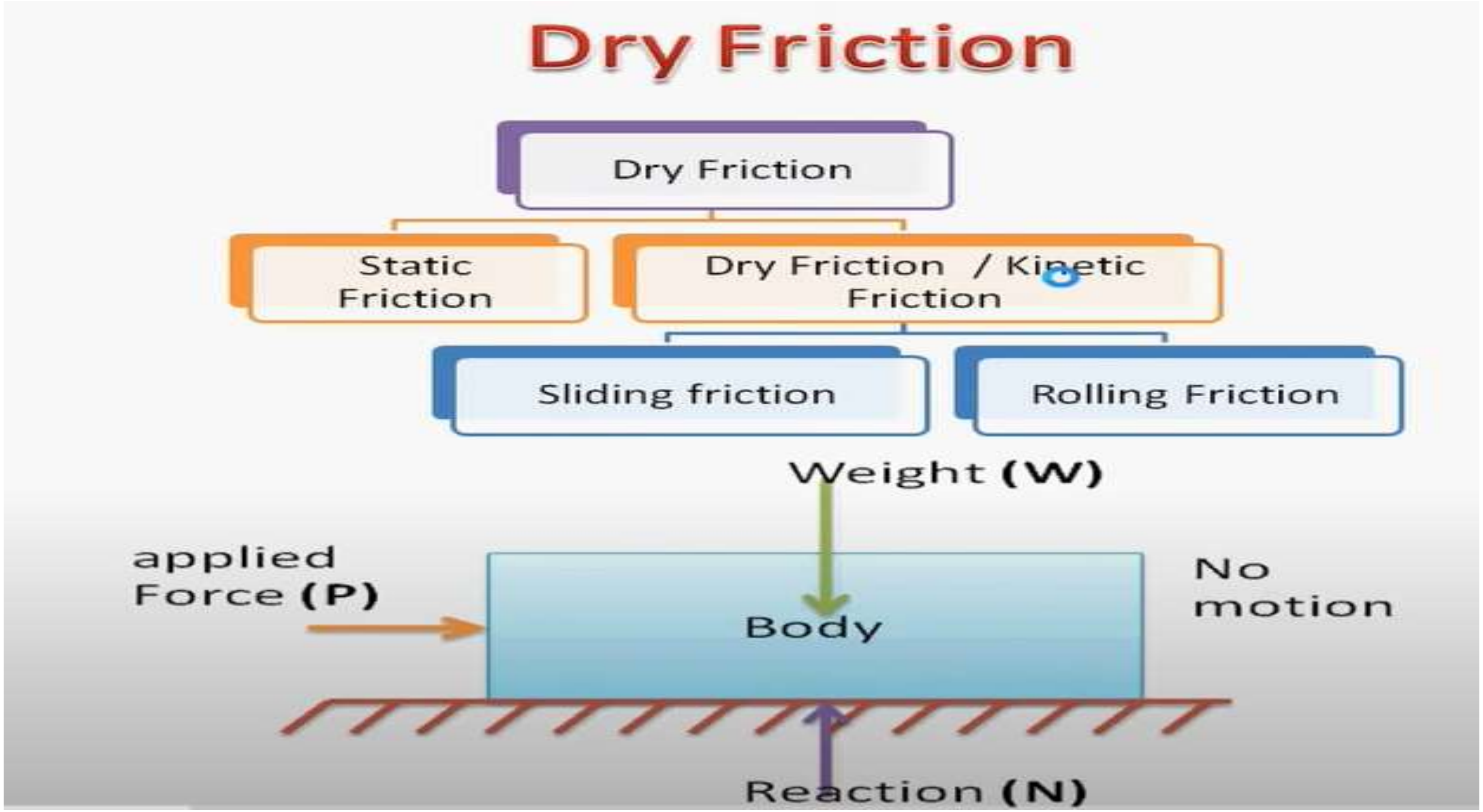


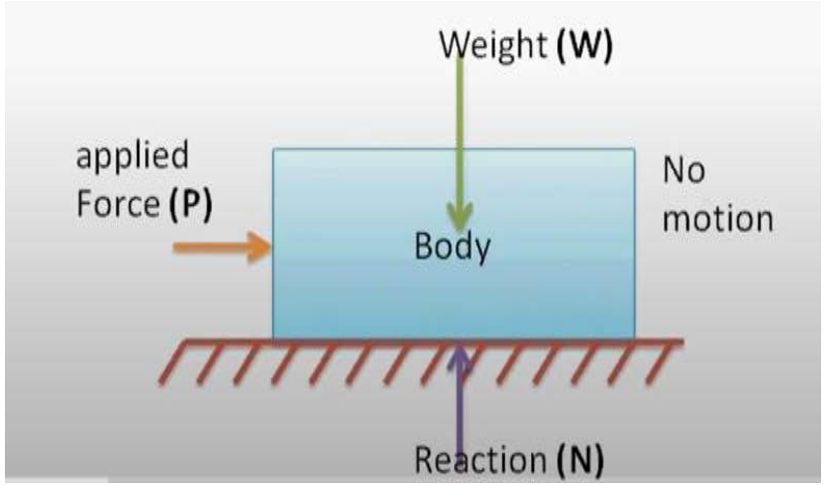
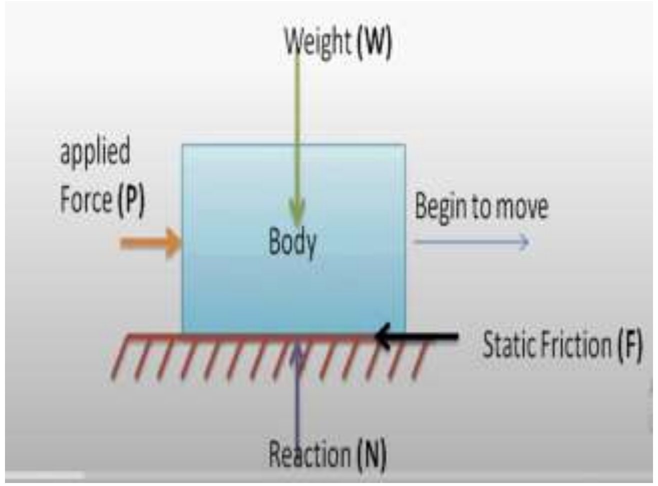
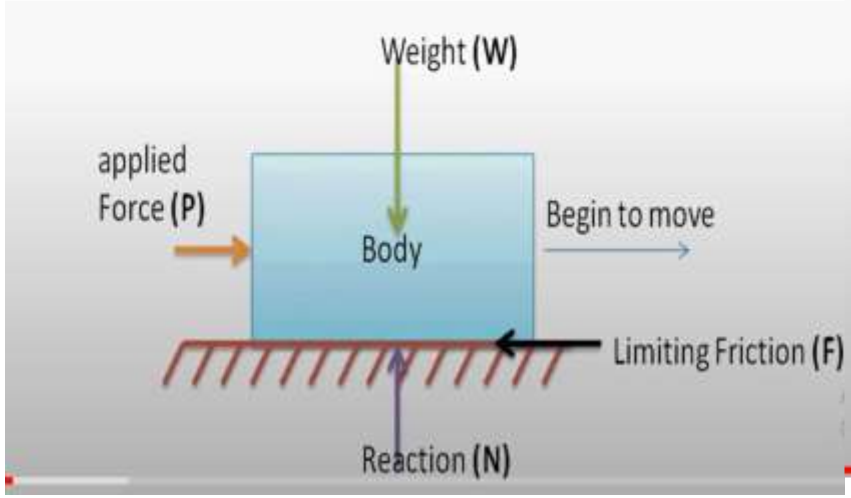
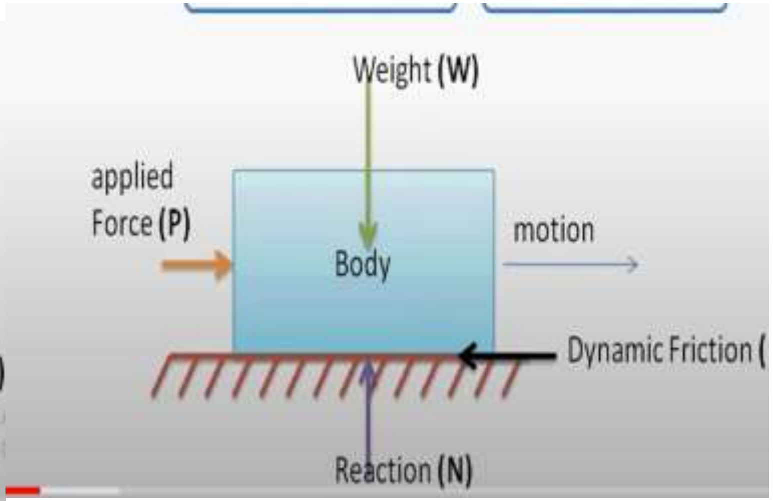


Advantages of friction:

- Friction enable us to write and draw on paper
- Car move due to friction
- Friction enables us to light a matchstick
- Friction enables us to walk without slipping
- To hold or grabbing something


1. LAW OF FRICTION
2.FORMULA OF FRICTION
3. ANGLE OF FRICTION & REPOSE


Dry Friction
Dry Friction
Static Friction
Dry Friction / Kinetic Friction
Sliding friction
Rolling Friction
Weight (W)
applied Force (P)
Body
No motion
Reaction (N)

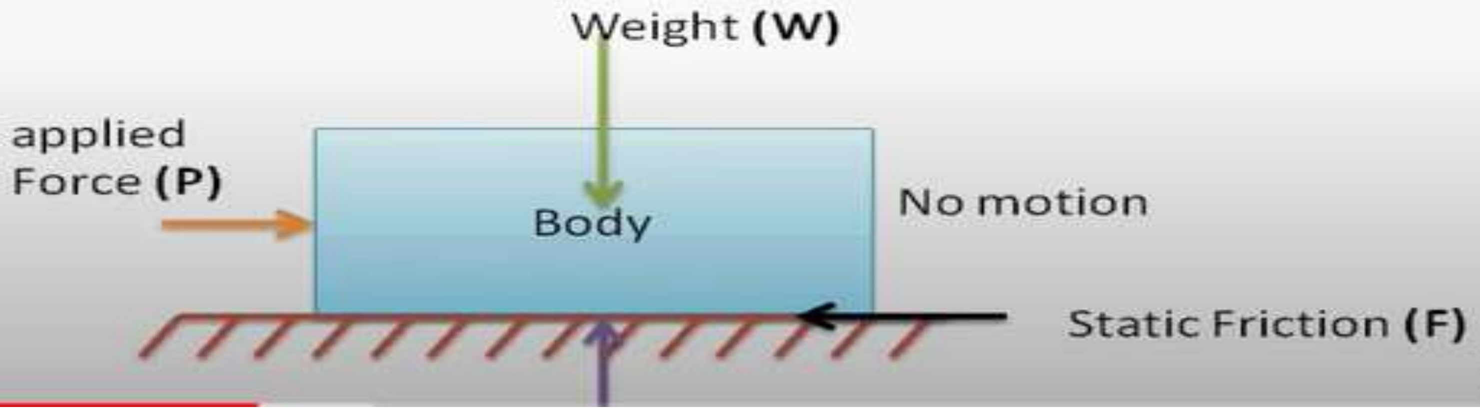

Weight (W)
applied
Force (P)
Body
No
motion
Reaction (N)

Weight (W)
applied
Force (P)
Body
Begin to move
Static Friction (F)
Reaction (N)

Weight (W)
applied
Force (P)
Body
Begin to move
Limiting Friction (F)
Reaction (N)

Weight (W)
applied
Force (P)
Body
motion
Dynamic Friction
Reaction (N)

Law of static friction ;

- The friction force always acts in a direction, opposite to that in which the body tends to move.

- The magnitude of friction force is equal to the external force.

$$F=P$$

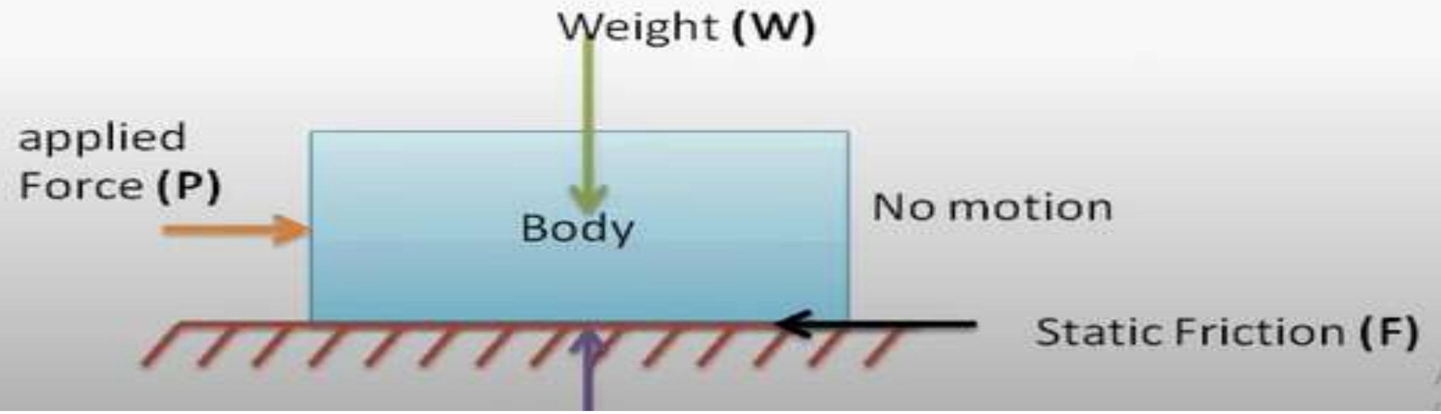


Law of static friction ;

- The ratio of limiting friction (F) and normal reaction (N) is constant.

  $F/N=\mu$

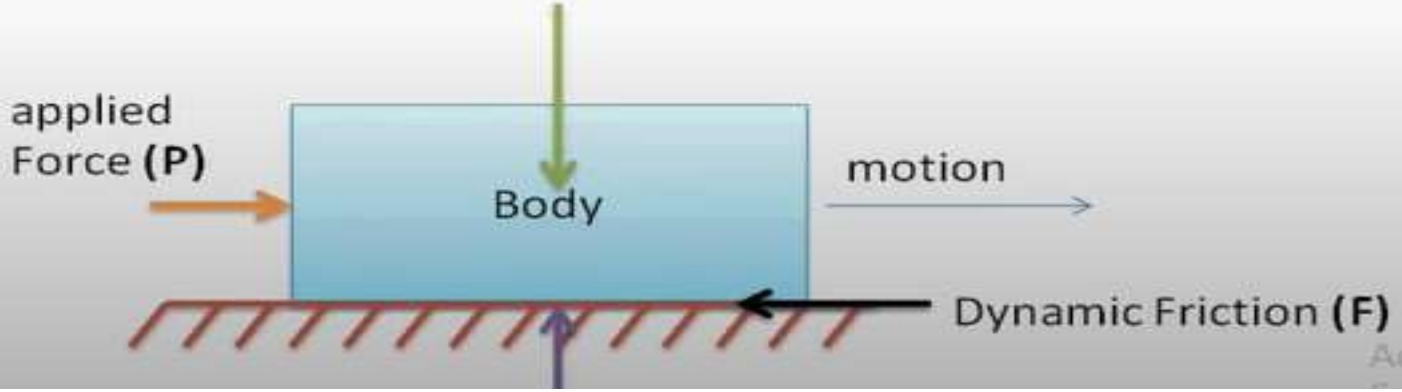
- The friction force does not depends upon the area of contact between the two surfaces.
- The friction force depends upon the roughness of the surfaces.



Activate Windo

Law of dynamic friction ;

- The friction force always acts in a direction, opposite that in which the body is moving.
- The ratio of limiting friction (F) and normal reaction (N) is constant & it is known as coefficient of friction (μ).
- For moderate speeds, the friction force remains constant. But it decreases slightly with the increase of speed.



Formula of limiting friction ;

According to law of friction,

Friction force α normal force

$F \alpha N$

$F = \mu N$

μ = coefficient of friction

- **The ratio of limiting friction (F) and normal reaction (N) is constant & it is known as coefficient of friction (μ).**

$\mu = F / N$

Angle of friction ;

- The angle between normal reaction and resultant force (R) is called angle of friction.

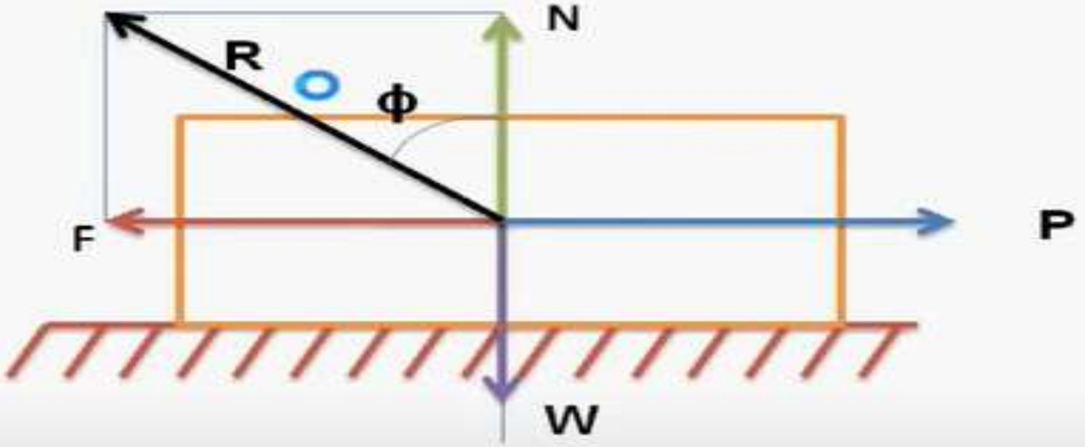
$\tan\phi = F/N$

ϕ = angle of friction

and

$\tan\phi = \mu$

μ = coefficient of friction



- It is also called limiting angle of friction.

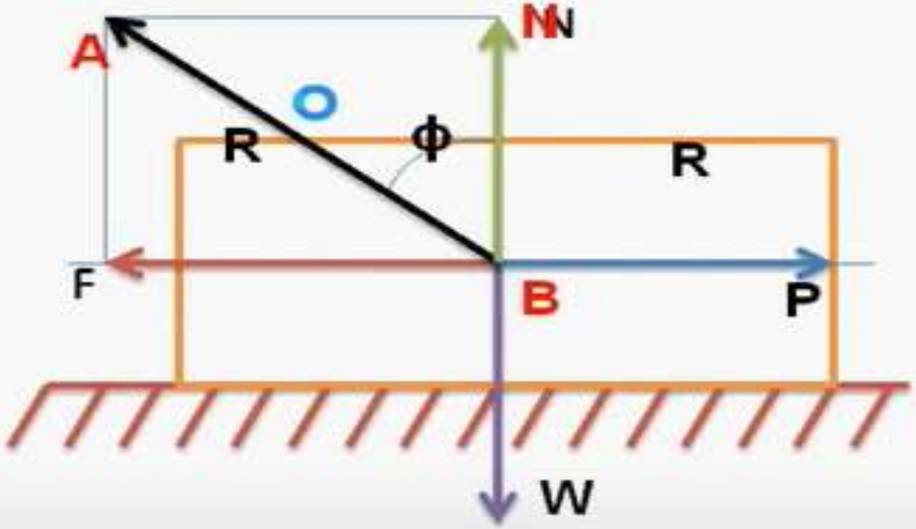
$\tan \phi = F/N$

ϕ = angle of friction

and

$\tan \phi = \mu$

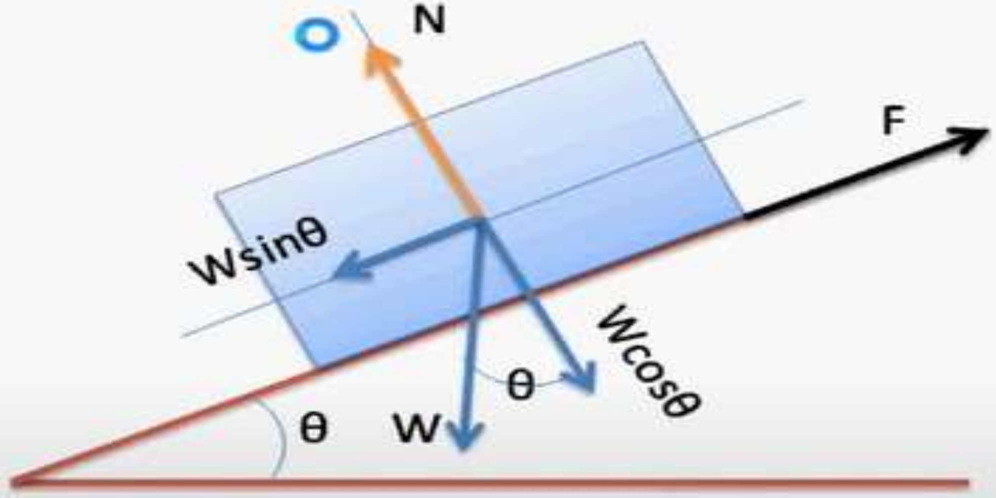
μ = coefficient of friction



Angle of repose:

- With increase in angle of the inclined surface, the maximum angle at which body starts sliding down is called angle of friction.

θ = angle of Repose



Week – 12 & 13

Lecture

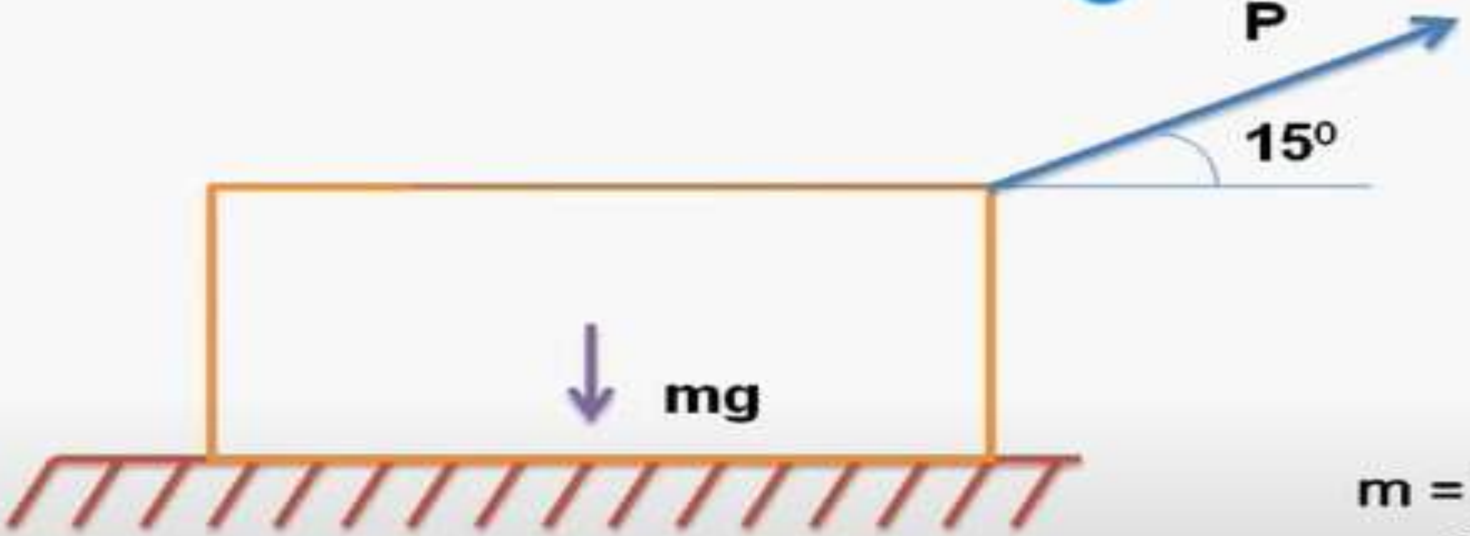
On

Friction

(139-159)

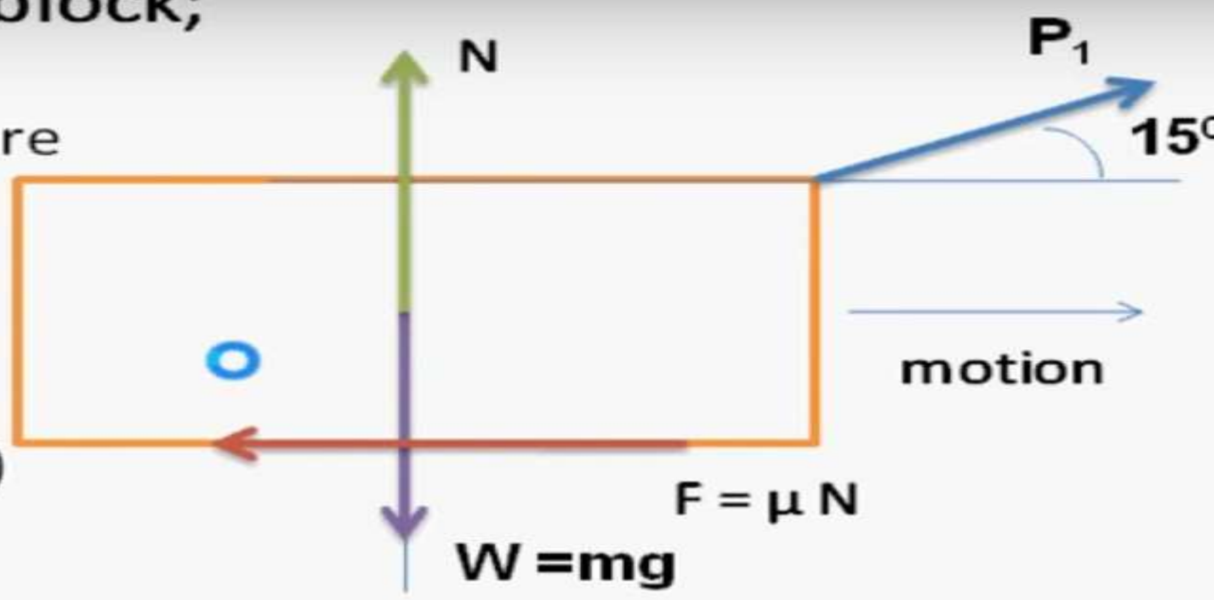
Problem-01

A wooden block rests on horizontal plane. Determine the minimum force required to (i) pull it (ii)push it . Assume the mass **m** of the block to be 5kg and the coefficient of friction μ is 0.4



$m = 5kg$
$\mu = 0.4$

(i) Minimum force required to pull block;



free body diagram of block is shown in figure

Writing the equation of equilibrium

$\Sigma F_x = 0$

$P_1 \cos 15 - \mu N = 0$

$0.966 P_1 - 0.4N = 0$ ....(i)

$\Sigma F_y = 0$

$P_1 \sin 15 + N - mg = 0$

$0.259 P_1 + N - 5 \times 9.81 = 0$

$0.259 P_1 + N = 49.05$ ......(ii)

From equation (i) & (ii)

$P_1 = 18.34$ N

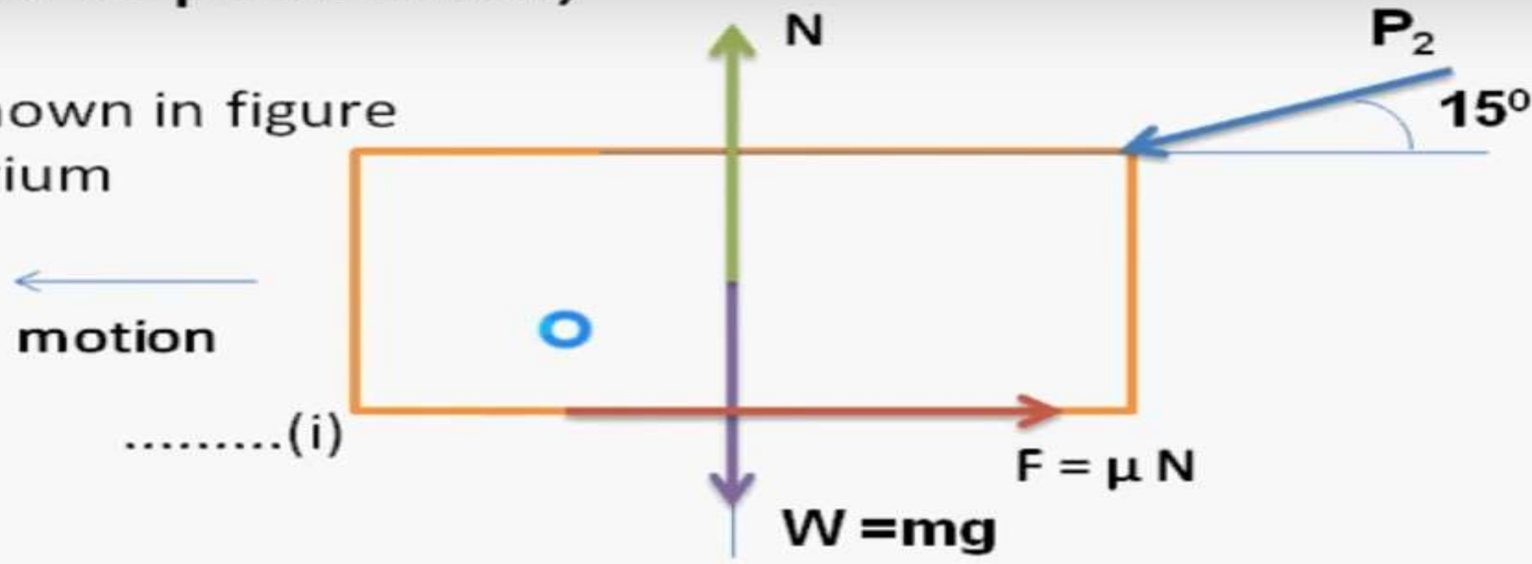
&

N = 44.30 N

minimum force required to pull block = **18.34 N**

(i) Minimum force required to push block;

free body diagram of block is shown in figure
Writing the equation of equilibrium



$\Sigma F_x = 0$

$-P_2 \cos 15 + \mu N = 0$

$-0.966 P_2 + 0.4N = 0$ ....(i)

$\Sigma F_y = 0$

$-P_2 \sin 15 + N - mg = 0$

$-0.259P_2 + N - 5 \times 9.81 = 0$

$-0.259P_2 + N = 49.05$ ......(ii)

From equation (i) & (ii)

$P_1 = 22.75$ N

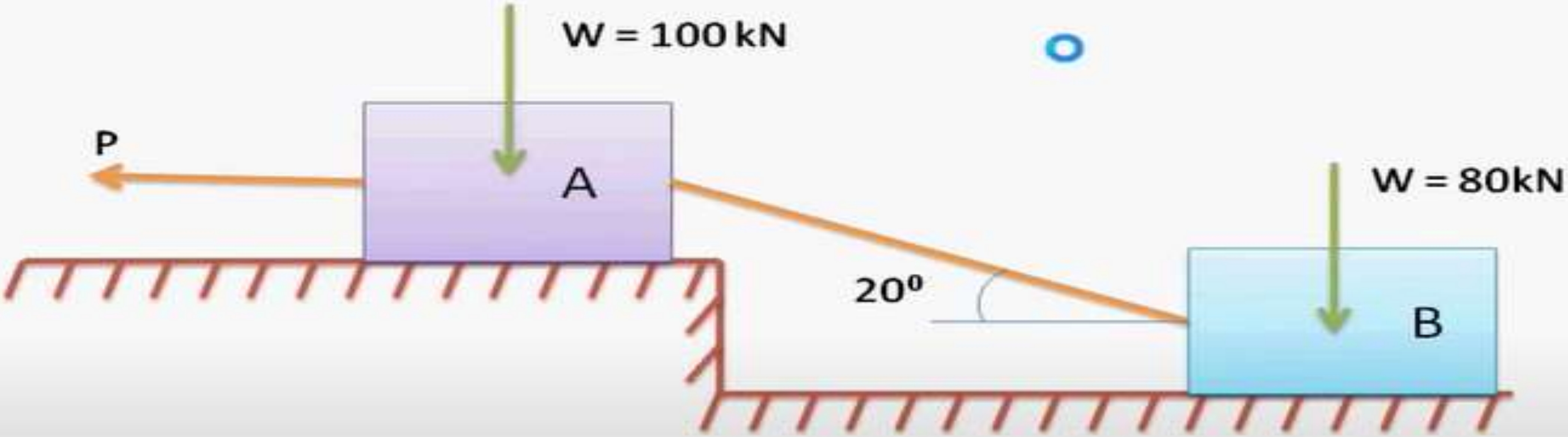
&

N = 54.94 N

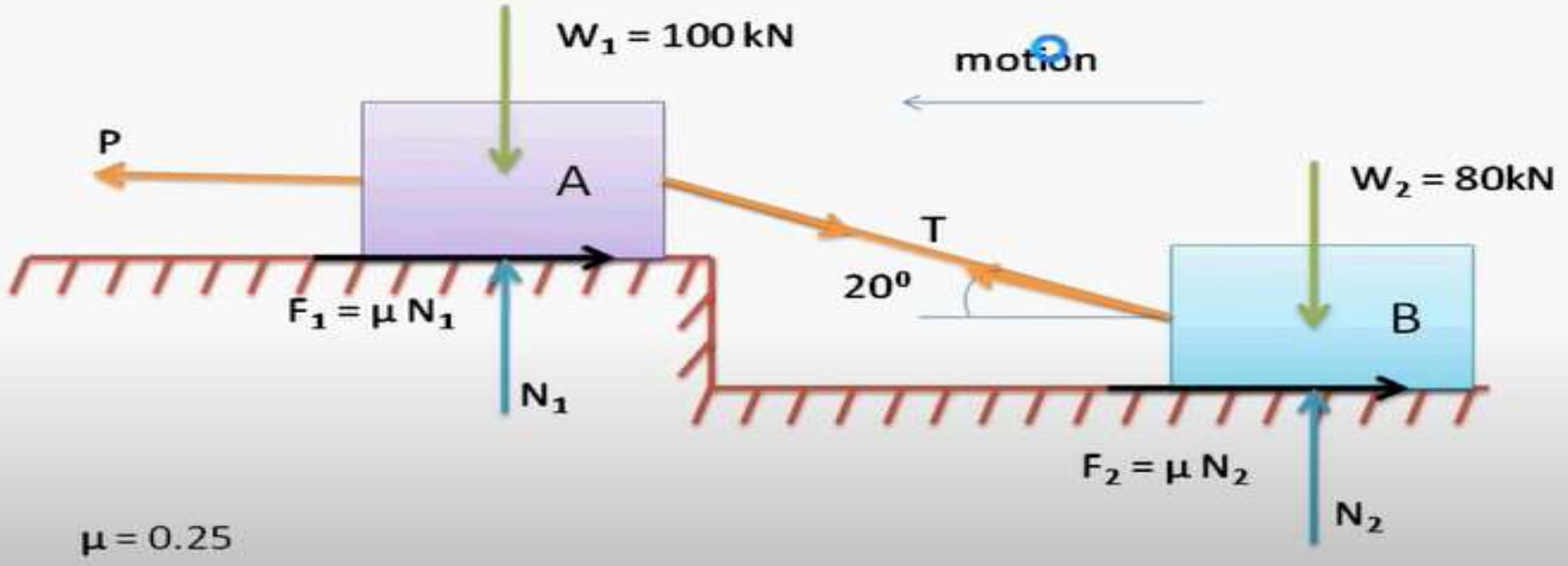
minimum force required to push block= **22.75 N**

Problem-02

Find the value of force P to just move the block. Assume coefficient of friction μ is 0.25 for all surfaces



Free body diagram of the block A & B



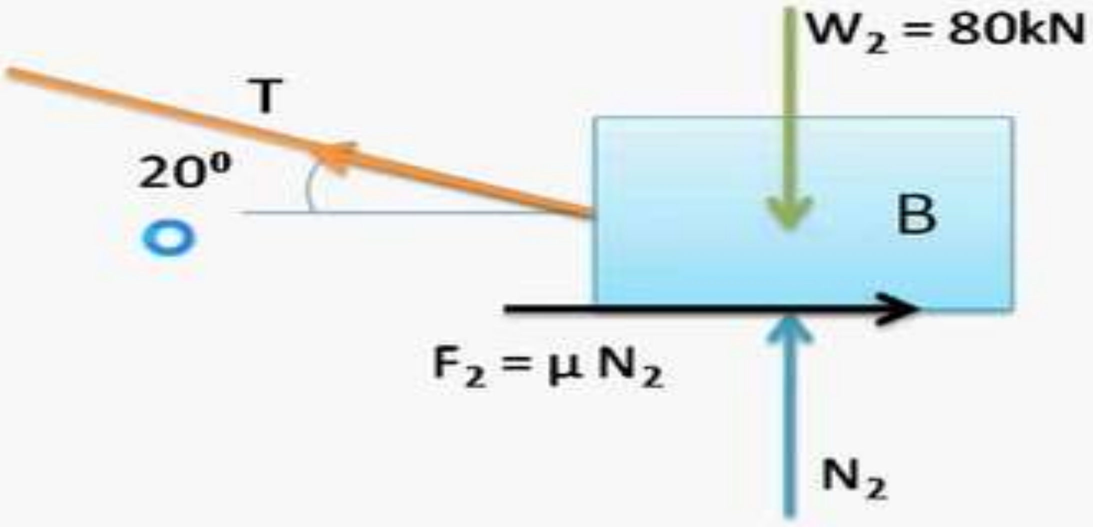
Consider Free body diagram of the block B;

Writing the equation of equilibrium

$\Sigma F_x = 0$

$-T\cos 20 + \mu N_2 = 0$

$-0.94\,T + 0.25\,N_2 = 0$ ....(i)



$\Sigma F_y = 0$

$T\sin 20 + N_2 - W_2 = 0$

$0.34T + N_2 - 80 = 0$

$0.34T + N_2 = 80$ ....(ii)

From equation (i) & (ii)

T = 19.51 kN

&

N_2 = 73.37 kN

Consider Free body diagram of the block A

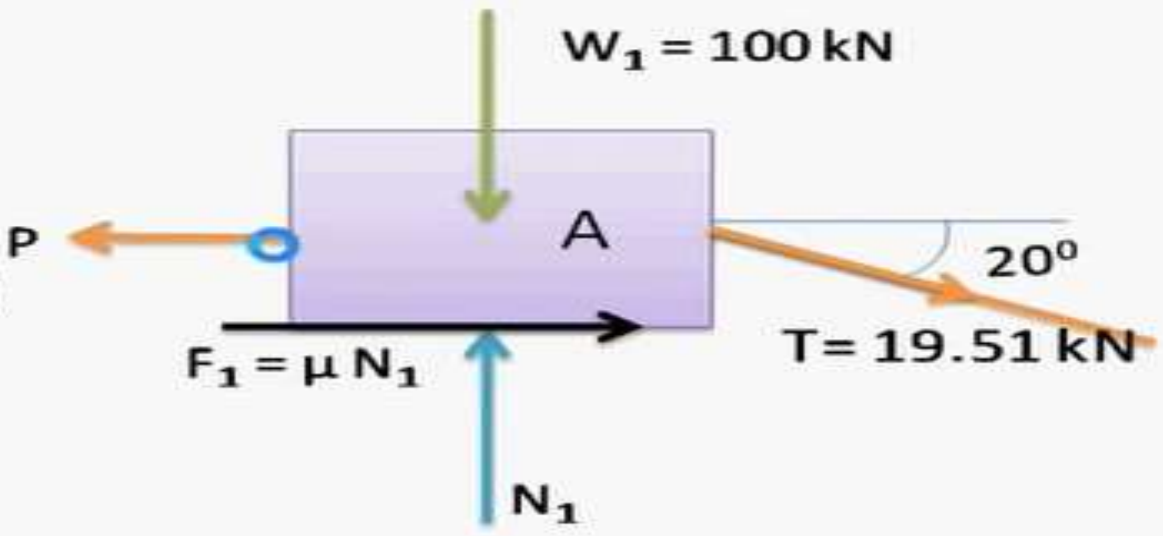
Writing the equation of equilibrium

$\Sigma F_x = 0$

$T \cos 20 + \mu N_1 - P = 0$

$0.94\,(19.51) + 0.25\,N_1 - P = 0$

$P - 0.25\,N_1 = 18.34$ ....(i)



$\Sigma F_y = 0$

$- T \sin 20 + N_1 - W_1 = 0$

$-0.34(19.51) + N_1 - 100 = 0$

$N_1 = 106.63$ kN

From equation (i)

P = 45 kN

value of force P to just move the block = 45 kN

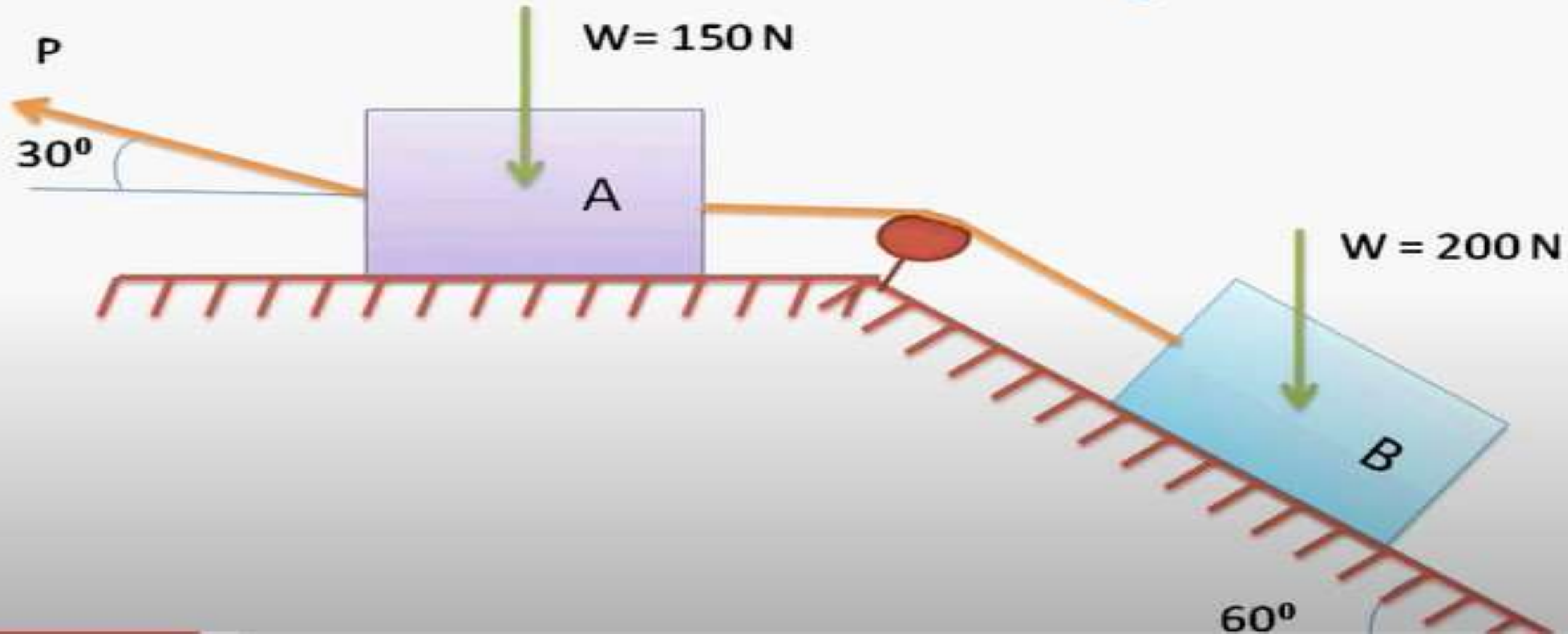
Problem-03

Find the value of force P to;

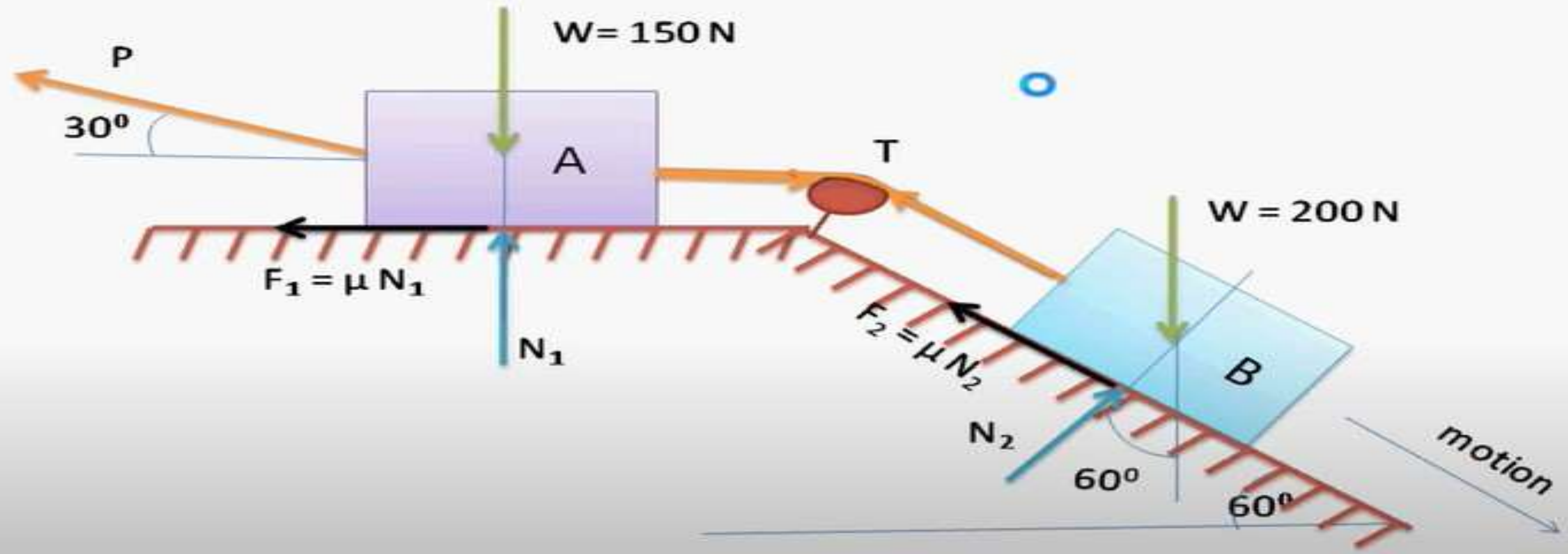
(i) just stop block B from moving down

(ii) just start block B to move up

Assume coefficient of friction μ is 0.2 for all surfaces.



(i) just stop block B from moving down:



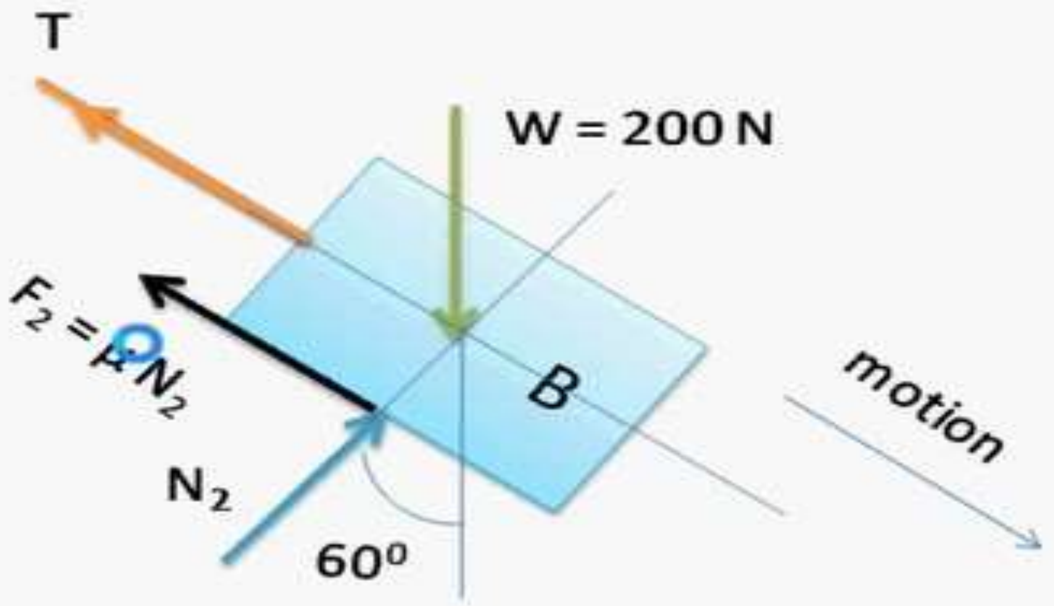
Consider FBD of block B;
Writing the equation of equilibrium

$\Sigma F_x = 0$

$W \sin 60 - T - \mu N_2 = 0$

$200 \sin 60 - T - 0.2 N_2 = 0$

$T + 0.2 N_2 = 173.20$ ....(i)



$\Sigma F_y = 0$

$- W \cos 60 + N_2 = 0$

$\mathbf{N_2 = 100\,N}$

From equation (i))

$\mathbf{T = 153.2\,N}$

Consider FBD of block A;

Writing the equation of equilibrium

$\Sigma F_x = 0$

$- P \cos 30 + T - \mu N_1 = 0$

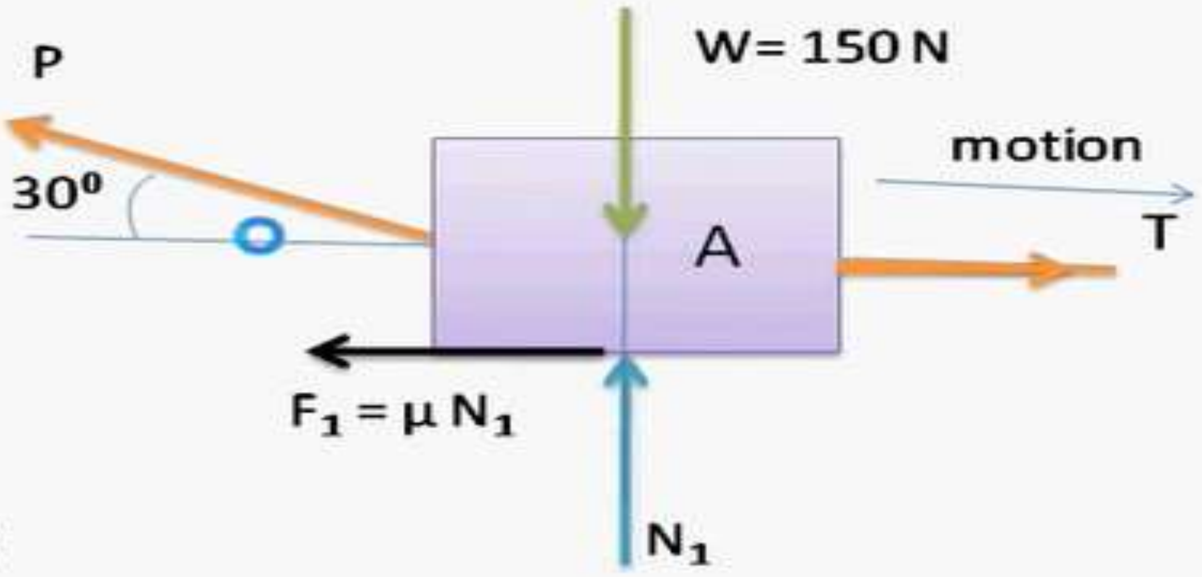
$-P \cos 30 + 153.2 - 0.2N_1 = 0$

$0.87P + 0.2N_1 = 153.2$ ....(i)

$\Sigma F_y = 0$

$P \sin 30 - W + N_1 = 0$

$0.5P + N_1 = 150$ ....(ii)

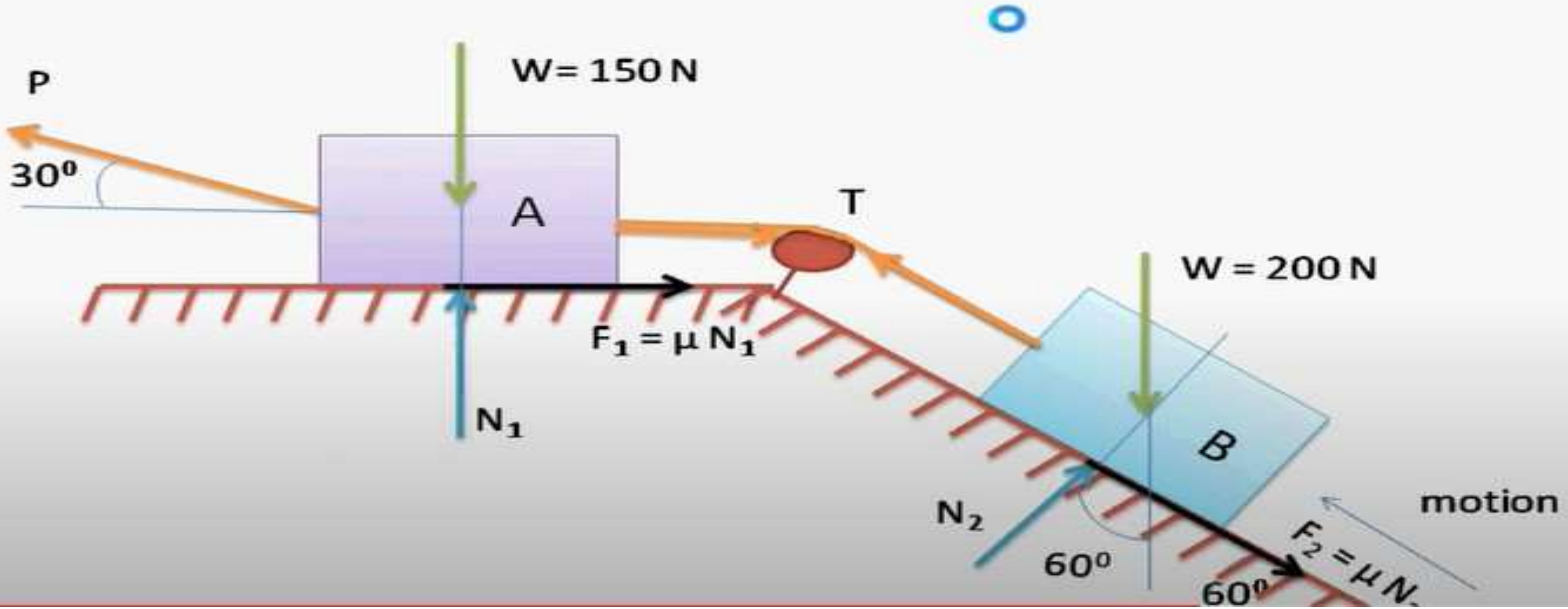


From equation (i) & (ii)

P = 160 N

$N_1 = 70$ N

(ii) just start block B to move up



Consider FBD of block B;

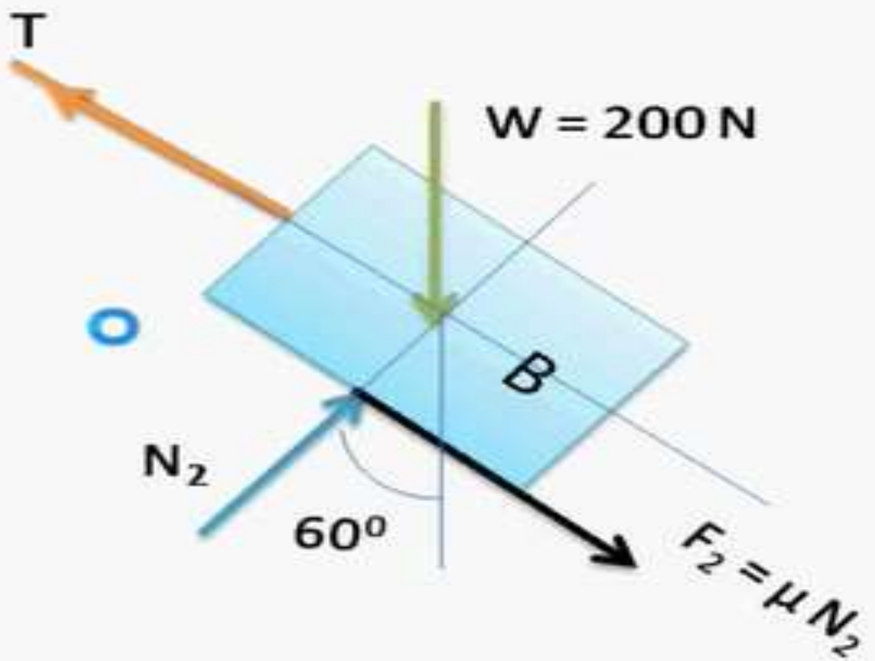
Writing the equation of equilibrium

$\Sigma F_x = 0$

$W \sin60 - T + \mu N_2 = 0$

$200 \sin60 - T + 0.2 N_2 = 0$

$T - 0.2 N_2 = 173.20$ ....(i)



$\Sigma F_y = 0$

$- W \cos60 + N_2 = 0$

$\mathbf{N_2 = 100\,N}$

From equation (i))

$\mathbf{T = 193.2N}$

Consider FBD of block A;

Writing the equation of equilibrium

$\Sigma F_x = 0$

$- P \cos 30 + T + \mu N_1 = 0$

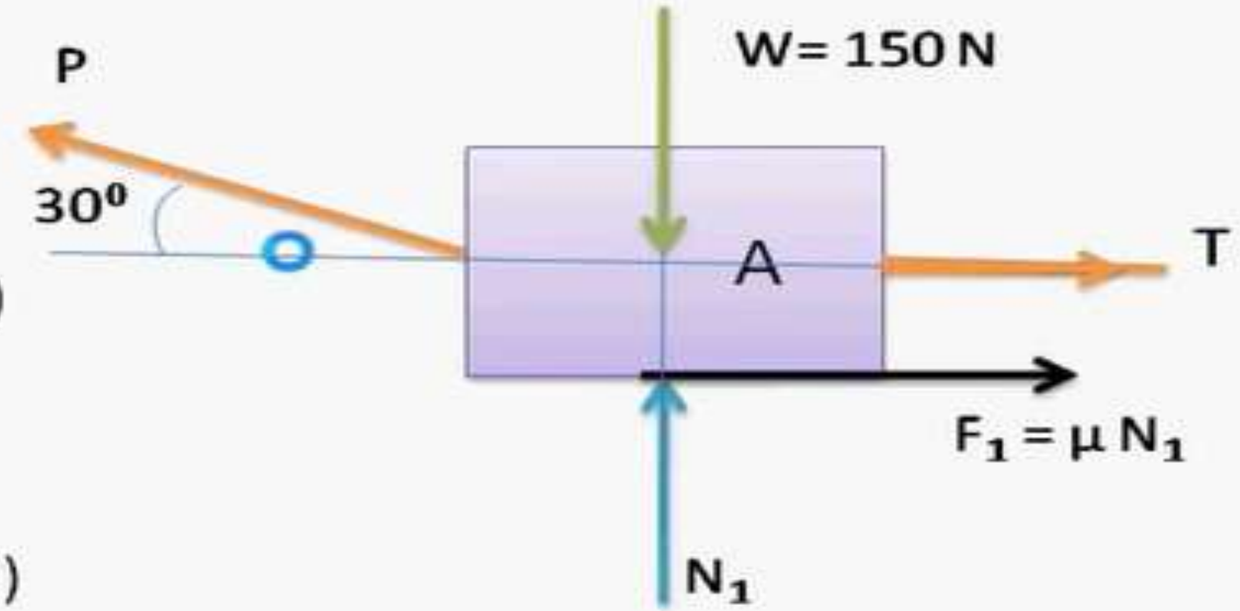
$-P \cos 30 + 193.2 + 0.2N_1 = 0$

$0.87P - 0.2N_1 = 193.2$ ....(i)

$\Sigma F_y = 0$

$P \sin 30 - W + N_1 = 0$

$0.5P + N_1 = 150$ ....(ii)



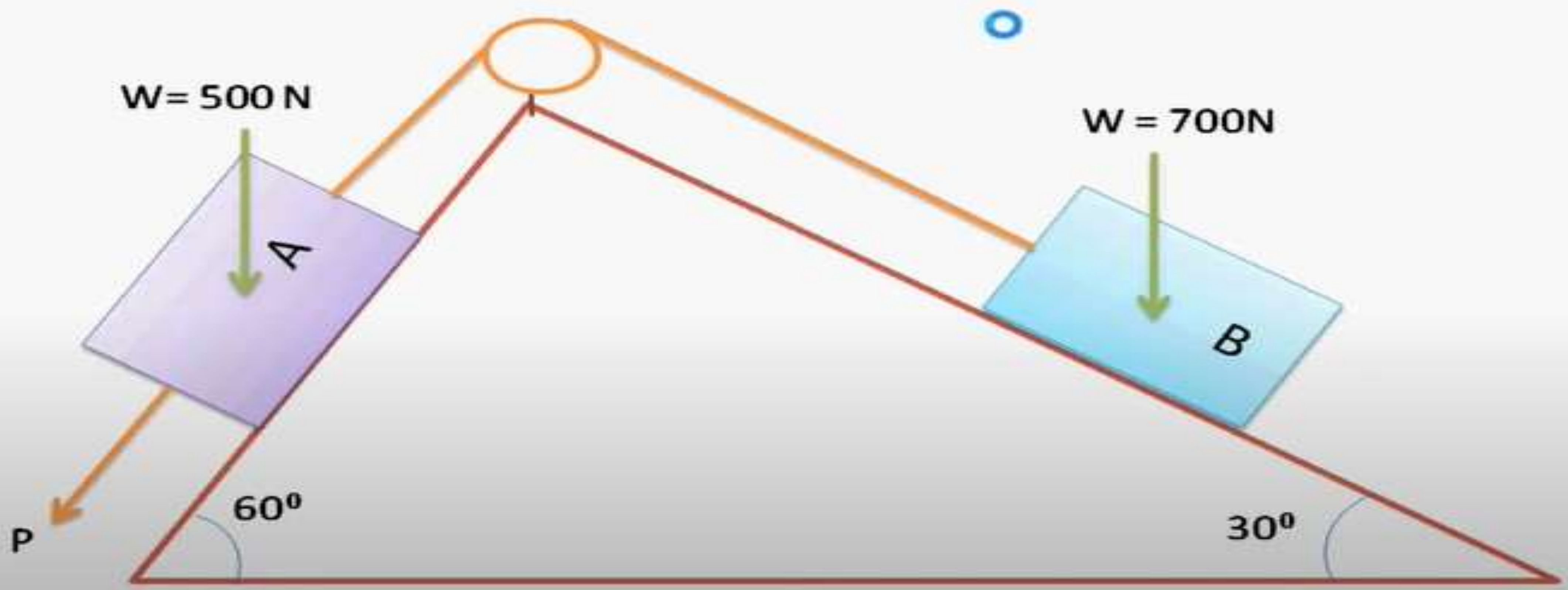
From equation (i) & (ii)

P = 230 N

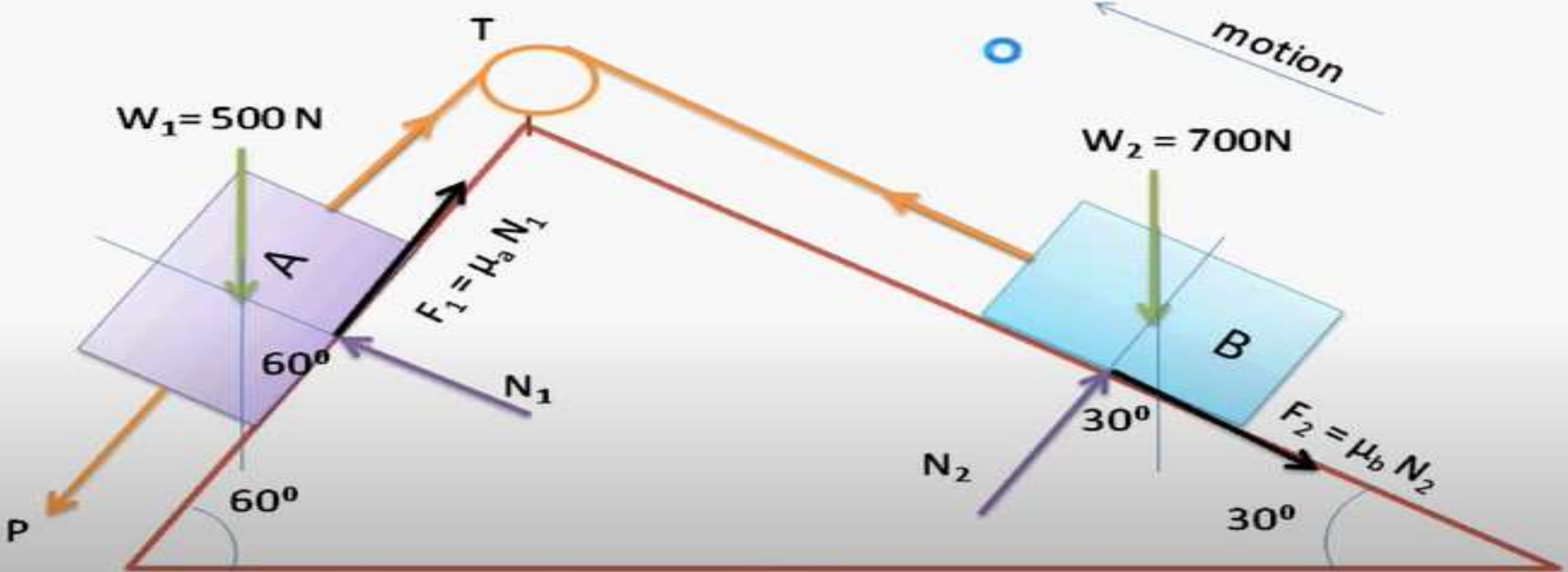
$N_1 = 34.95$ N

Problem-04

If $\mu_a = 0.2$ and $\mu_b = 0.25$ Find the value of force P to just pull the block B.



FBD of block A & B:



Consider FBD of block B:

Writing the equation of equilibrium

$\Sigma F_x = 0$

$W_2 \sin30 - T + \mu_b N_2 = 0$

$700 \sin30 - T + 0.25 N_2 = 0$

$T - 0.25 N_2 = 350$ ....(i)

$\Sigma F_y = 0$

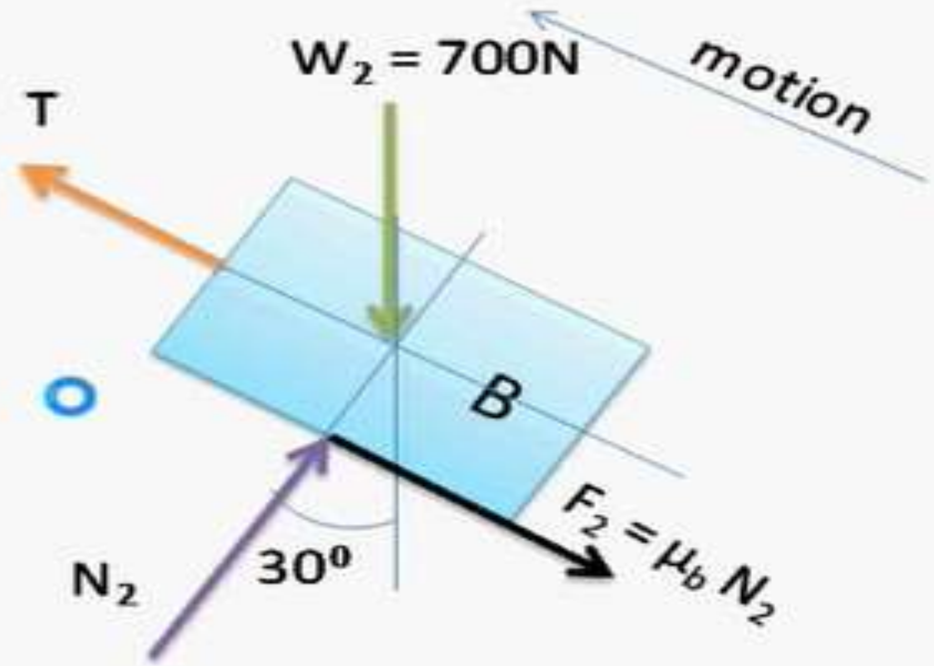
$- W_2 \cos30 + N_2 = 0$

$- 700 \cos30 + N_2 = 0$

$\mathbf{N_2 = 606.22\,N}$

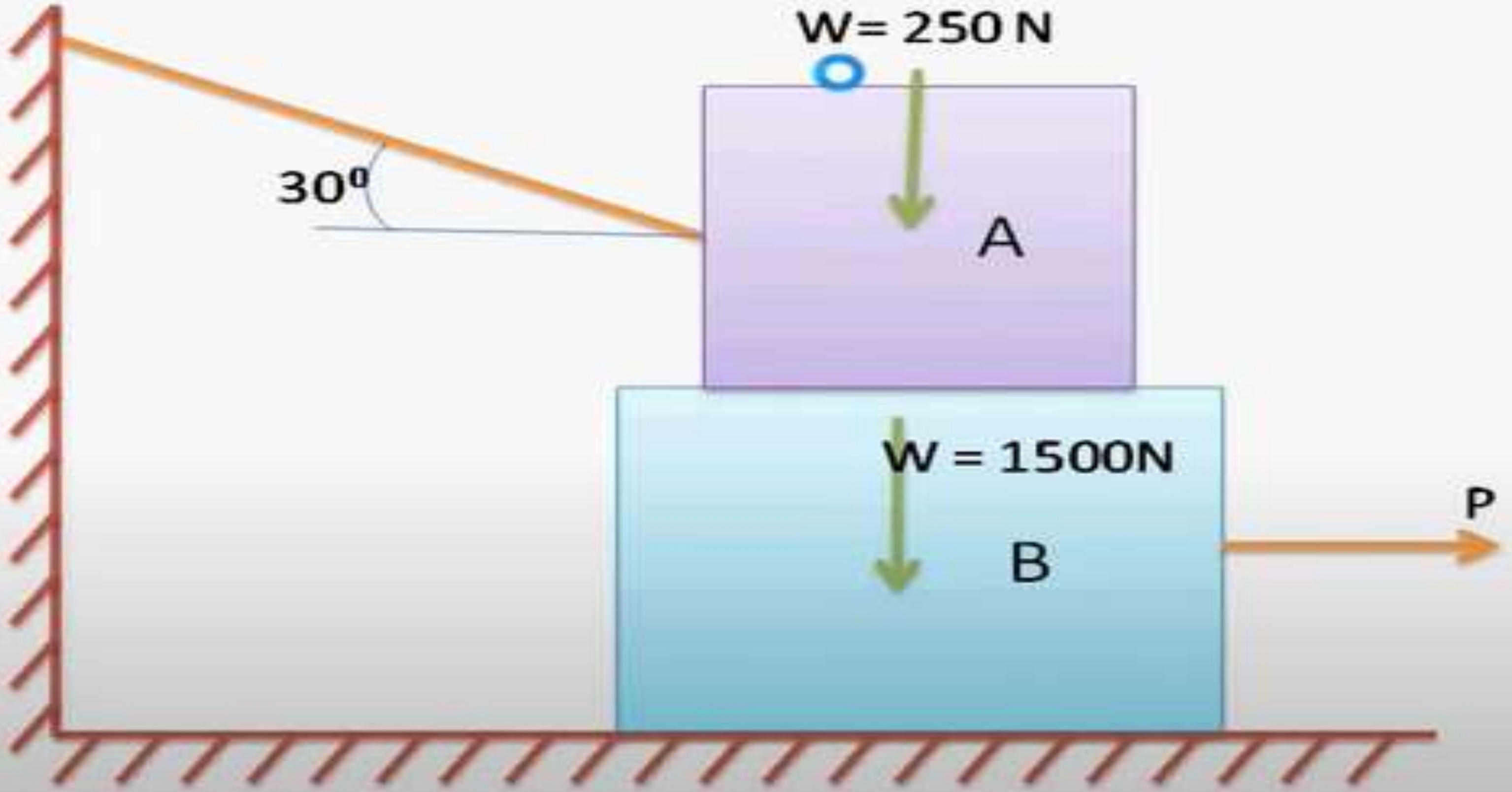
From equation (i)

$\mathbf{T = 501.56\,N}$

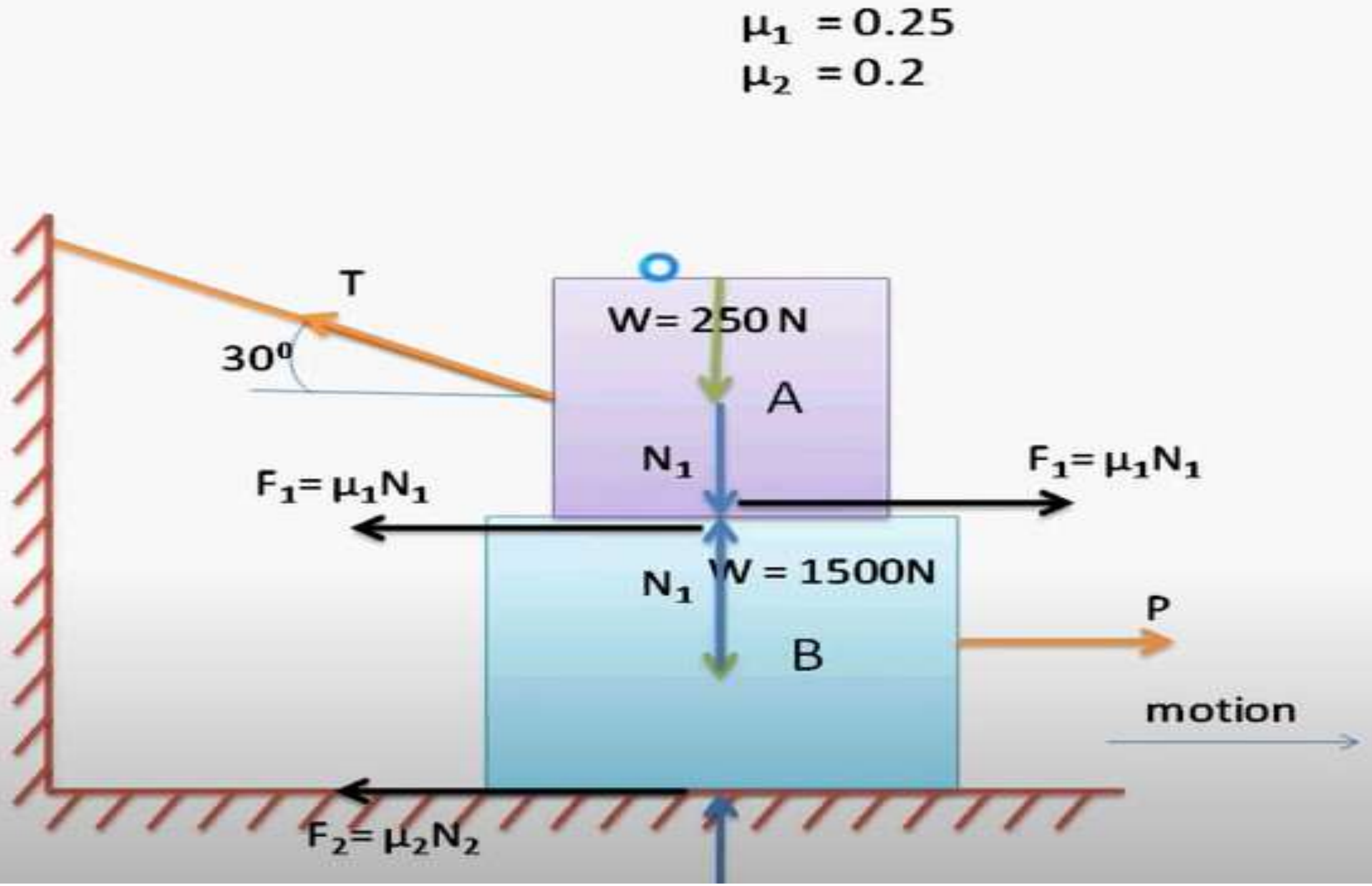


Problem-05

Find the value of force P to just pull the block B. If coefficient of friction between block A & B and between block B & horizontal surface is 0.25 and 0.2 respectively.



Free body diagram of Block A & B:



Consider Free body diagram of Block A :

Writing the equation of equilibrium

$\Sigma F_x = 0$

$-T\cos 30 + \mu_1 N_1 = 0$

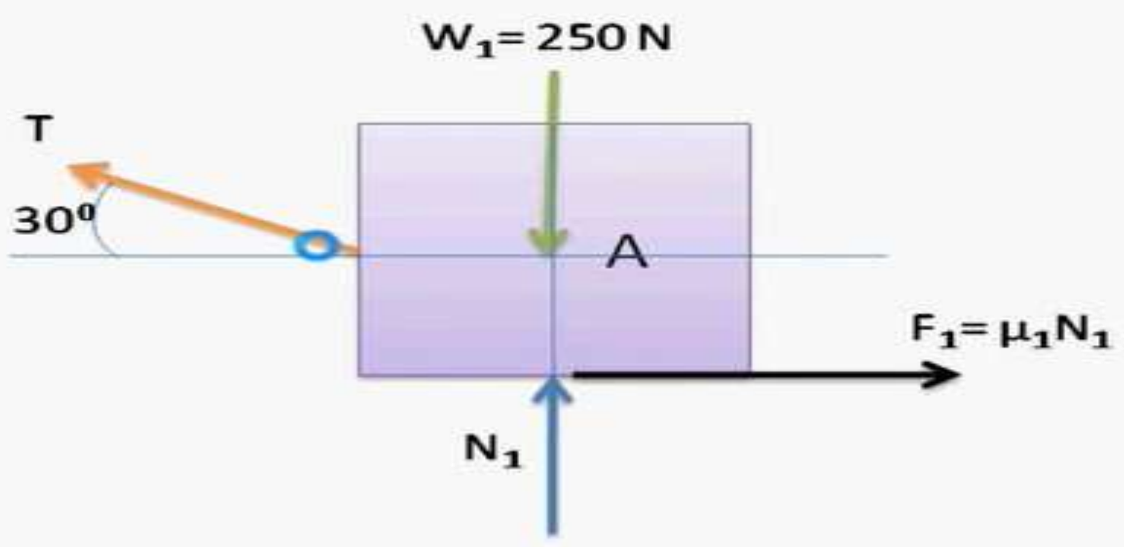
$-0.866\,T + 0.25 N_1 = 0$ ....(i)

$\Sigma F_y = 0$

$T\sin 30 - W_1 + N_1 = 0$

$0.5\,T - 250 + N_1 = 0$

$0.5\,T + N_1 = 250$ ....(ii)



From equation (i) & (ii)

T = 63.07 N

N_1 = 218.47 N

Consider Free body diagram of Block B:

Writing the equation of equilibrium

$\Sigma F_x = 0$

$P - \mu_1 N_1 - \mu_2 N_2 = 0$

$P - (0.25 \times 218.47) - 0.2N_2 = 0$

$P - 0.2N_2 = 54.62$ ....(i)

$\Sigma F_y = 0$

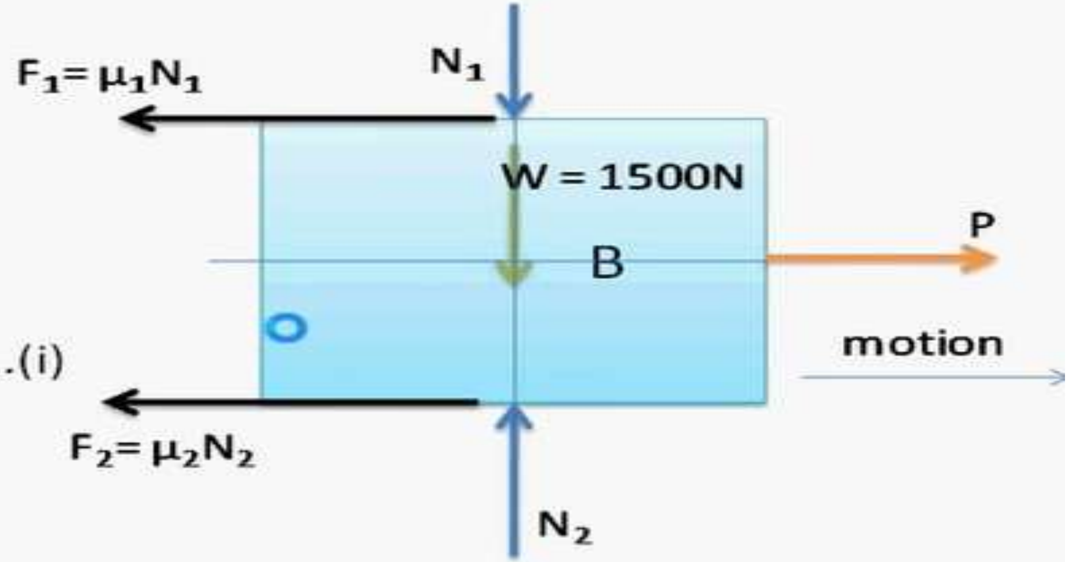
$- W_2 - N_1 + N_2 = 0$

$-1500 - 218.47 + N_2 = 0$

$N_2 = 1718.47$ N

From equation (i)

P = 398.31 N



Week – 14

Lecture

On

Friction

(161-177)

3.3 APPLICATIONS OF FRICTION

1.Ladder friction

2.Wedge friction

3.Screw friction.

LADDER FRICTION

The ladder is a device for climbing or scaling on the roofs or walls. It consists of two long uprightsof wood, iron or rope connected by a number of crosspieces called rungs. These running serve as steps.Consider a ladder AB resting on the rough ground and leaning against a wall, as shown in figure 3.9.

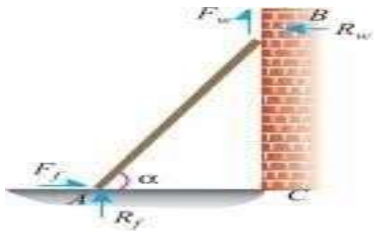


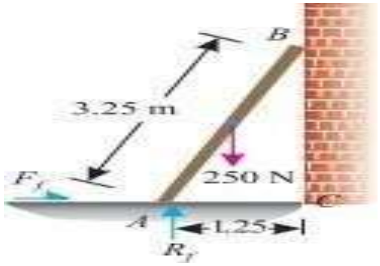
Fig 3.13

As the upper end of the ladder tends to slip downwards, thereforethe direction of the force of friction between the ladder and the wall (Fw) will be upwards as shown in the figure. Similarly, as the lowerend of the ladder tends to slip away from the wall, therefore the directionof the force of friction between the ladder and the floor (F_f) will betowards the wall as shown in the figure.Since the system is in equilibrium, therefore the algebraic sumof the horizontal and vertical components of the forces must also beequal to zero.

Note: The normal reaction at the floor (R_f) will act perpendicular of the floor. Similarly, normal reaction of the wall (Rw) will also act perpendicular to the wall.

Example 3.6 A uniform ladder of length 3.25 m and weighing 250 N is placed against a smooth vertical wall with its lower end 1.25 m from the wall. The coefficient of friction between theladder and floor is 0.3.What is the frictional force acting on the ladder at the

point of contact between the ladder andthefloor? Show that the ladder will remain in equilibrium in this position.



Solution. Given: Length of the ladder (l) = 3.25 m; Weight of the ladder (w) = 250 N;
Distance between the lower end of ladder and wall = 1.25 m and coefficient of friction between theladder and floor (μf) = 0.3.
Frictional force acting on the ladder. The forces acting on the ladder.
LetFf= Frictional force acting on the ladder at thePoint of contact between the ladder and floor, and
Rf = Normal reaction at the floor.
Since the ladder is placed against a smooth vertical wall, therefore there will be no friction at the point of contact between the ladder and wall.

Resolving the forces vertically, Rf= 250 N
From the geometry of the figure, we find that

BC = $\sqrt{(3.25) - (1.25)} = 3.0$ m

Taking moments about B and equating the same,
Ff× 3 = (Rf× 1.25) – (250 × 0.625) = (250 × 1.25) – 156.3 = ~~156.2 N~~

Ff = 52.1 N
Equilibrium of the ladder

We know that the maximum force of friction available at the point of contact between the ladder and the floor
μRf= 0.3 × 250 = 75 N
Thus, we see that the amount of the force of friction available at the point of contact (75 N) is more than the force of friction required for equilibrium (52.1 N). Therefore, the ladder will remain
inan equilibrium position.

Example 3.7 A ladder 5 meters long rests on a horizontal ground and leans against asmooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900

N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750N stands on a rung 1.5metre from the bottom of the ladder.

Calculate the coefficient of friction between the ladder and the floor.

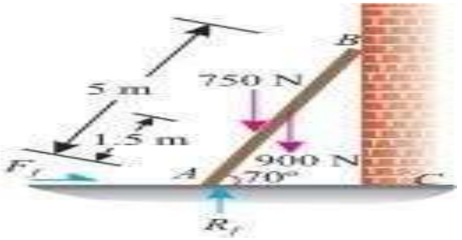


Fig 3.15

Solution. Given: Length of the ladder (l) = 5 m; Angle which the ladder makes with the horizontal (α) = 70°; Weight of the ladder (w1) = 900 N; Weight of man (w2) = 750 N and distancebetween the man and bottom of ladder = 1.5 m.

Forces acting on the ladder are shown in Fig.

Let μf = Coefficient of friction between ladder and floor and

R_f = Normal reaction at the floor. Resolving the forces vertically, R_f = 900 + 750 = 1650 N ...(i)

∴ Force of friction at A

$F_f = \mu_f \times R_f = \mu_f \times 1650$ ...(ii)

Now taking moments about B, and equating the same,

$R_f \times 5 \sin 20° = (F_f \times 5 \cos 20°) + (900 \times 2.5 \sin 20°) + (750 \times 3.5 \sin 20°)$

$= (F_f \times 5 \cos 20°) + (4875 \sin 20°)$

$= (\mu_f \times 1650 \times 5 \cos 20°) + 4875 \sin 20°$

and now substituting the values of R_f and F_f from equations (i) and (ii) $1650 \times 5 \sin 20° = (\mu_f \times 1650 \times 5 \cos 20°) + (4875 \sin 20°)$

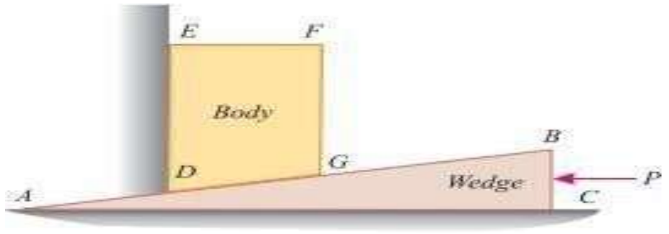
Dividing both sides by 5 sin 20°, $1650 = (\mu_f \times 1650 \cot 20°) + 975$

$= (\mu_f \times 1650 \times 2.7475) + 975 = 4533\,\mu_f + 975$

∴ $\mu = 0.15$

WEDGE FRICTION:

A wedge is, usually, of a triangular or trapezoidal in cross-section. It is, generally, used forslightadjustments in the position of a body i.e. for tightening fits or keys for shafts. Sometimes, awedge is also used for lifting heavy weights as shown in fig.3.16



It will be interesting to know that the problems on wedges are basically the problems ofequilibrium on inclined planes. Thus, these problems may be solved either by the equilibrium method or by applying Lami‟s theorem. Now consider a wedge ABC, which is used to lift the body DEFG.

Let W = Weight fo the body DEFG,

P = Force required to lift the body, and

μ = Coefficient of friction onthe planes AB, AC and DE such that tan φ = μ.

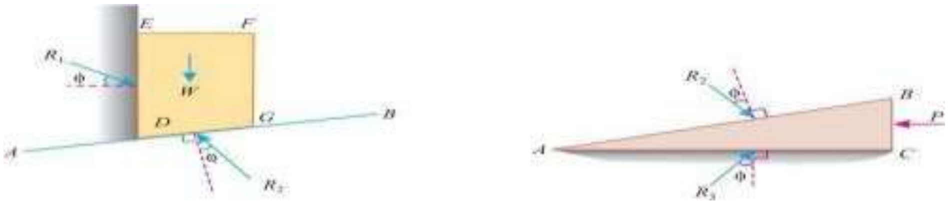
It will be interesting to know that the problems on wedges are basically the problems of equilibrium on inclined planes. Thus, these problems may be solved either by the equilibrium method or by applying Lami‟s theorem. Now consider a wedge ABC, which is used to lift the body DEFG.

Let W = Weight of the body DEFG,

P = Force required to lift the body, and

μ = Coefficient of friction onthe planes AB, AC and DE such that tan φ = μ.

A little consideration will show that when the force is sufficient to lift the body, the slidingwill take place along three planes AB, AC and DE will also occur as shown in Fig. 3.17 (a) and (b).



The three reactions and the horizontal force (P) may now be found out either by graphical method or analytical method as discussed below:

GRAPHICAL METHOD

1. First of all, draw the space diagram for the body DEFG and the wedge ABC as shown inFig.3.18 (a). Now draw the reactions R_1, R_2 and R_3 at angle f with normal to the faces DE, AB and AC respectively (such that tan φ = μ).

2. Now consider the equilibrium of the body DEFG. We know that the body is in equilibrium- under the action of

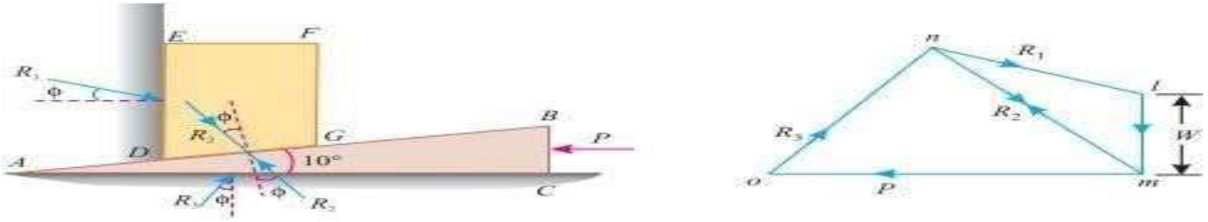
(a)Its own weight (W) acting downwards

(b)Reaction R_1 on the face DE, and

(c)Reaction R_2 on the face AB.

Now, in order to draw the vector diagram for the above mentioned three forces, takesome suitable point l and draw a vertical line lm parallel to the line of action of the weight(W) and cut off lm equal to the weight of the body to some suitable scale. Through l drawa line parallel to the reaction R_1.

Similarly, through m draw a line parallel to the reactionR_2, meeting the first line at n as shown in Fig. 3.18(b).



(a) Space diagram (b) Vector diagram

3. Now consider the equilibrium of the wedge ABC. We know that it is equilibrium underthe action of

(a)Force acting on the wedge (P),

(b)Reaction R_2 on the face AB, and

(c)Reaction R_3 on the face AC.

Now, in order to draw the vector diagram for the above mentioned three forces, through mdraw a horizontal line parallel to the force (P) acting on the wedge. Similarly, through ndraw a line parallel to the reaction R_3 meeting the first line at O as shown in Fig.3.18 (b).

4. Now the force (P) required on the wedge to raise the load will be given by mo. to thescale.

EXAMPLE 3.8 A block weighing 1500 N, overlying a 10° wedge on a horizontal floor andleaning against a vertical wall, is to be raised by applying a horizontal force to the

wedge. Assuming the coefficient of friction between all the surface in contact to be 0.3, Determine the minimum horizontal force required to raise the block.

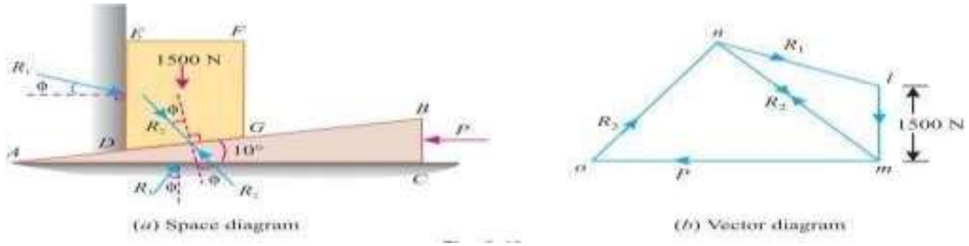


Fig 3.19

Solution. Given: Weight of the block (W) = 1500 N; Angle of the wedge (α) = 10° and coefficient of friction between all the four surfaces of contact (μ) = 0.3 = tan φ or φ = 16.7°.

P = Minimum horizontal force required to raise the block.

The example may be solved graphically or analytically. But we shall solve it by both the method

Graphical method

1. First of all, draw the space diagram for the block DEFG and the wedge ABC as shown in Fig.3.19 (a). Now draw reactions R_1, R_2 and R_3 at angles of φ (i.e. 16.7° with normal to the faces DE, AB and AC respectively.

2. Take some suitable point l, and draw vertical line lm equal to 1500 N to some suitable scale (representing the weight of the block). Through l, draw a line parallel to the reaction R_1. Similarly, through m draw another line parallel to the reaction R_2 meeting the first line at n.

3. Now through m, draw a horizontal line (representing the horizontal force P). Similarly,through n draw a line parallel to the reaction R3 meeting the first line at O as shown in Fig.(b).

4. Now measuring mo to the scale, we find that the required horizontal force

P = 1420 N. Ans.

Week – 15

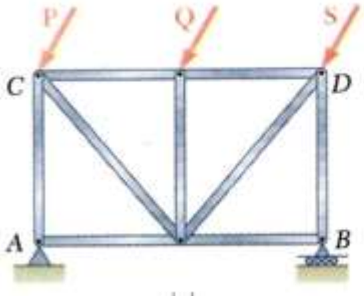
Lecture

On

Simple Truss Problems

(186-195)

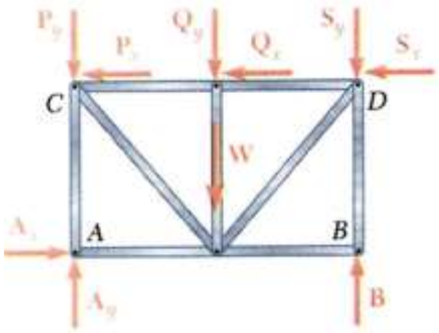
Equilibrium of rigid body



Equations of equilibrium become

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

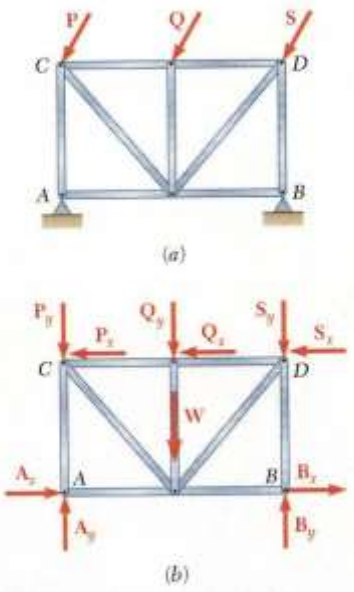
There are three unknowns and number of equation is three.



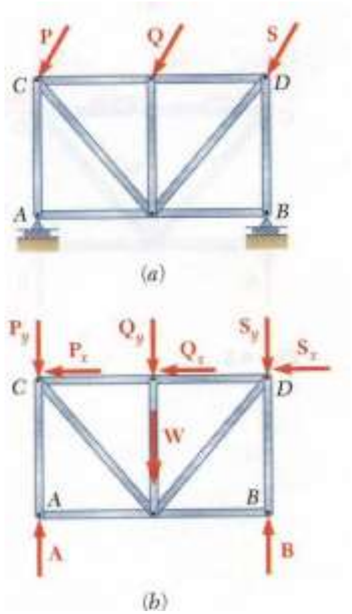
Therefore, the structure is statically determinate

The rigid body is completely constrained

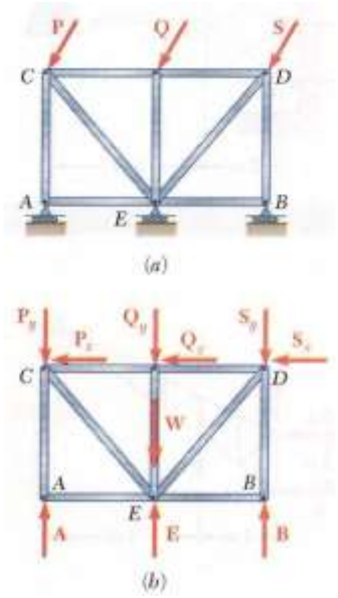
Equilibrium of rigid body



More unknowns than equations



Fewer unknowns than equations, partially constrained



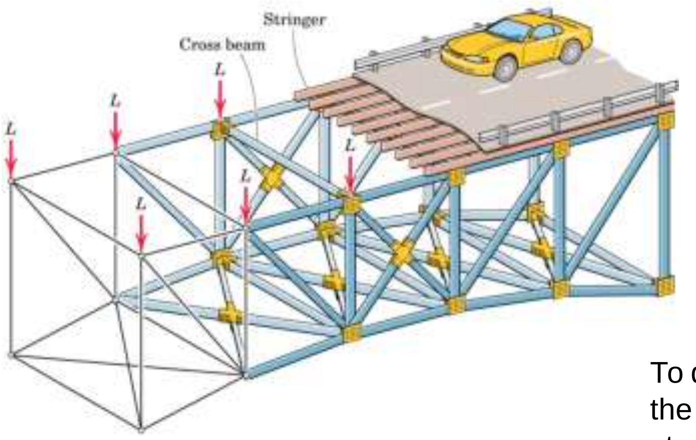
Equal number unknowns and equations but improperly constrained

Structural Analysis

Engineering Structure

An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it.

Structural Analysis



Statically Determinate Structures

To determine the internal forces in the structure, dismember the structure and analyze separate free body diagrams of individual members or combination of members.

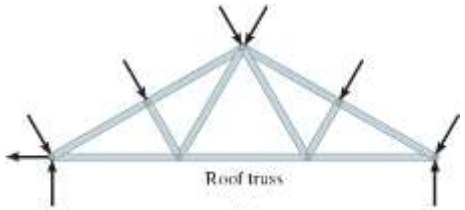
Structural Analysis: Plane Truss

Truss: A framework composed of members joined at their ends to form a rigid structure

Joints (Connections): Welded, Riveted, Bolted, Pinned

Plane Truss: Members lie in a single plane

(a)

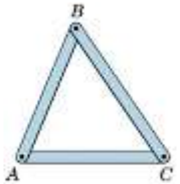


(b)

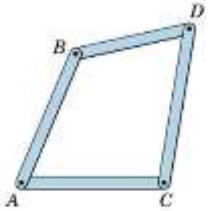
Structural Analysis: Plane Truss

Simple Trusses

Basic Element of a Plane Truss is the Triangle



- Three bars joined by pins at their ends □ Rigid Frame
  - Non-collapsible and deformation of members due to induced internal strains is negligible

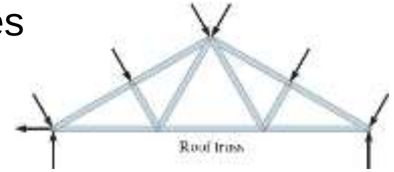


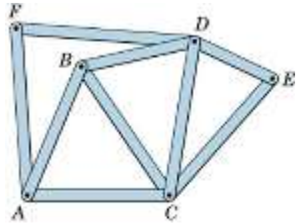
- Four or more bars polygon □ Non-Rigid Frame

  How to make it rigid or stable?

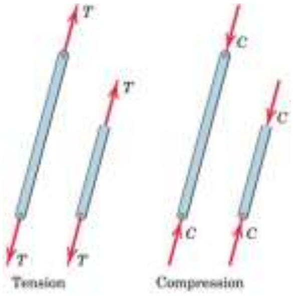
  by forming more triangles!

Structures built from basic triangles
□Simple Trusses





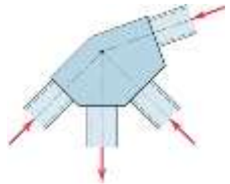
Structural Analysis: Plane Truss



Two-Force Members

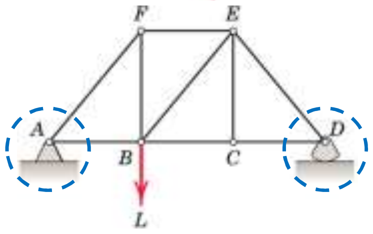
Basic Assumptions in Truss Analysis

- ➢ All members are two-force members.
- ➢ Weight of the members is small compared with the force it supports (weight may be considered at joints)
- ➢ No effect of bending on members even if weight is considered



- ➢ External forces are applied at the pin connections
- ➢ Welded or riveted connections
- ➢ Pin Joint if the member centerlines are concurrent at the joint

Common Practice in Large Trusses



- ➢ Roller/Rocker at one end. Why?
  - ➢ to accommodate deformations due to temperature changes and applied loads.
  - ➢ otherwise it will be a statically indeterminate truss

Structural Analysis: Plane Truss

Truss Analysis: Method of Joints

- Finding forces in members

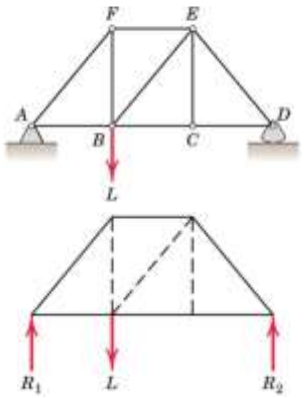
Method of Joints: Conditions of equilibrium are satisfied for the forces at each joint

– Equilibrium of concurrent forces at each joint

–only two independent equilibrium equations are involved

Steps of Analysis

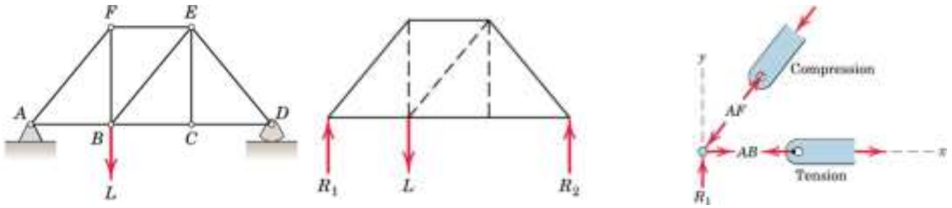
1. Draw Free Body Diagram of Truss
2. Determine external reactions by applying equilibrium equations to the whole truss
3. Perform the force analysis of the remainder of the truss by Method of Joints



Structural Analysis: Plane Truss

Method of Joints

- Start with any joint where at least one known load exists and where not more than two unknown forces are present.



FBD of Joint A and members AB and AF: Magnitude of forces denoted as *AB* & *AF*

- Tension indicated by an arrow away from the pin
- Compression indicated by an arrow toward the pin

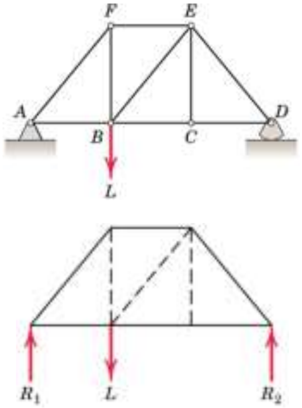
Magnitude of AF from $\sum F_y = 0$

Magnitude of AB from $\sum F_x = 0$

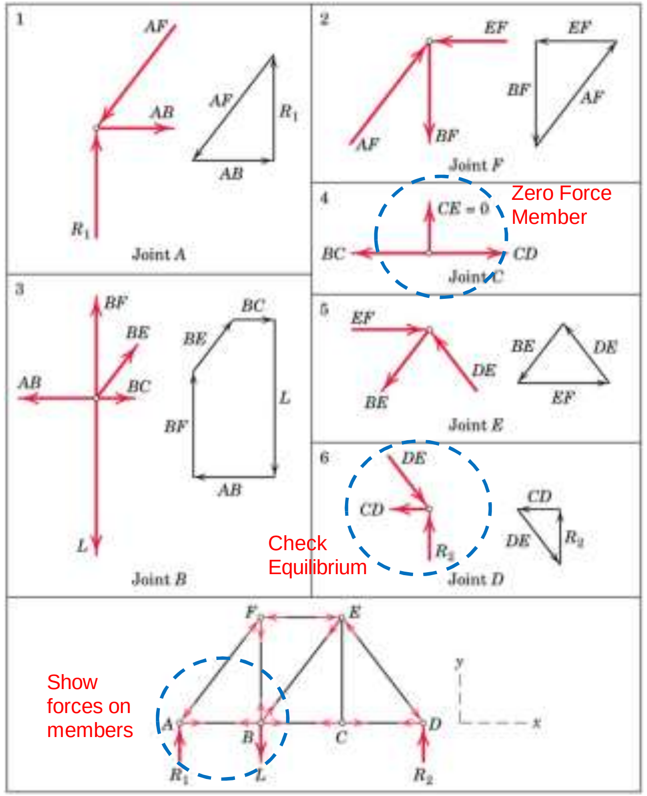
Analyze joints F, B, C, E, & D in that order to complete the analysis

Structural Analysis: Plane Truss

Method of Joints



- Negative force if assumed sense is incorrect



Week – 16

Lecture

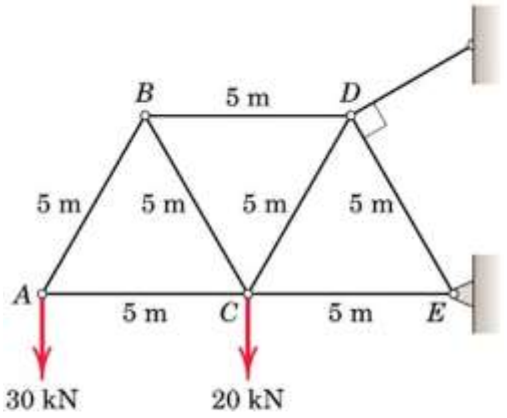
On

Simple Truss Problems

(179-197)

Method of Joints: Example

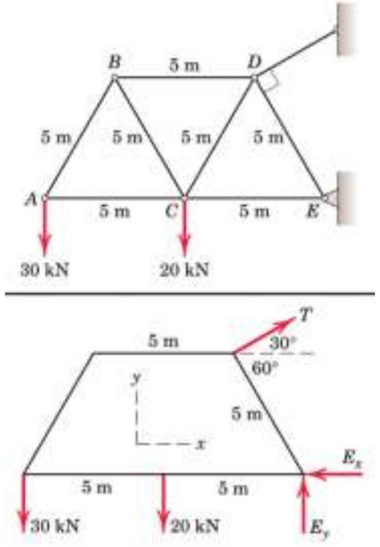
Determine the force in each member of the loaded truss by Method of Joints



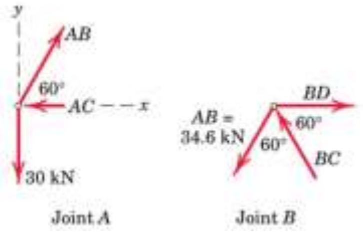
Method of Joints: Example

Solution

$[\Sigma M_E = 0]$	$5T - 20(5) - 30(10) = 0$	$T = 80$ kN
$[\Sigma F_x = 0]$	$80 \cos 30° - E_x = 0$	$E_x = 69.3$ kN
$[\Sigma F_y = 0]$	$80 \sin 30° + E_y - 20 - 30 = 0$	$E_y = 10$ kN

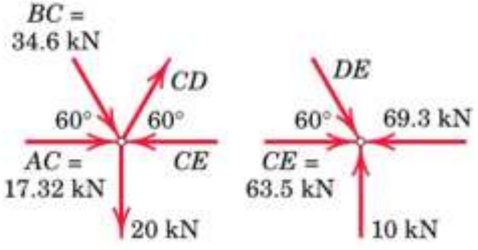


$[\Sigma F_y = 0]$	$0.866AB - 30 = 0$	$AB = 34.6$ kN T
$[\Sigma F_x = 0]$	$AC - 0.5(34.6) = 0$	$AC = 17.32$ kN C
$[\Sigma F_y = 0]$	$0.866BC - 0.866(34.6) = 0$	$BC = 34.6$ kN C
$[\Sigma F_x = 0]$	$BD - 2(0.5)(34.6) = 0$	$BD = 34.6$ kN T

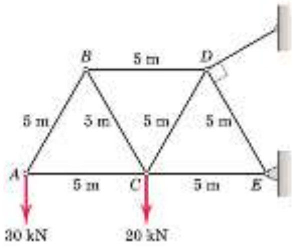


Method of Joints: Example

Solution



Joint C Joint E



$[\Sigma F_y = 0]$ $0.866CD - 0.866(34.6) - 20 = 0$

$CD = 57.7 \text{ kN } T$

$[\Sigma F_x = 0]$ $CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

$CE = 63.5 \text{ kN } C$

$[\Sigma F_y = 0]$ $0.866DE = 10$ $DE = 11.55 \text{ kN } C$

and the equation $\Sigma F_x = 0$ checks.

Structural Analysis: Plane Truss

When more number of members/supports are present than are needed to prevent collapse/stability

☐ Statically Indeterminate Truss

• cannot be analyzed using equations of equilibrium alone!
•additional members or supports which are not necessary for maintaining the equilibrium configuration ☐ Redundant

Internal and External Redundancy

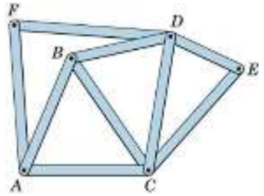
Extra Supports than required ☐ External Redundancy

– Degree of indeterminacy from available equilibrium equations

Extra Members than required ☐ Internal Redundancy

(truss must be removed from the supports to calculate internal redundancy)

– Is this truss statically determinate internally?



Truss is statically determinate internally if $m + 3 = 2j$

$m = 2j - 3$ m is number of members, and j is number of joints in truss

Structural Analysis: Plane Truss

Internal Redundancy or
Degree of Internal Static Indeterminacy
Extra Members than required ☐ Internal Redundancy

Equilibrium of each joint can be specified by two scalar force equations ☐ 2j equations for a truss with "j" number of joints
☐ Known Quantities

For a truss with "m" number of two force members, and maximum 3 unknown support reactions ☐ Total Unknowns = m + 3
("m" member forces and 3 reactions for externally determinate truss)
Therefore:
$m + 3 = 2j$ ☐ Statically Determinate Internally
$m + 3 > 2j$ ☐ Statically Indeterminate Internally
$m + 3 < 2j$ ☐ Unstable Truss

A necessary condition for Stability but not a sufficient condition since one or more members can be arranged in such a way as not to contribute to stable configuration of the entire truss

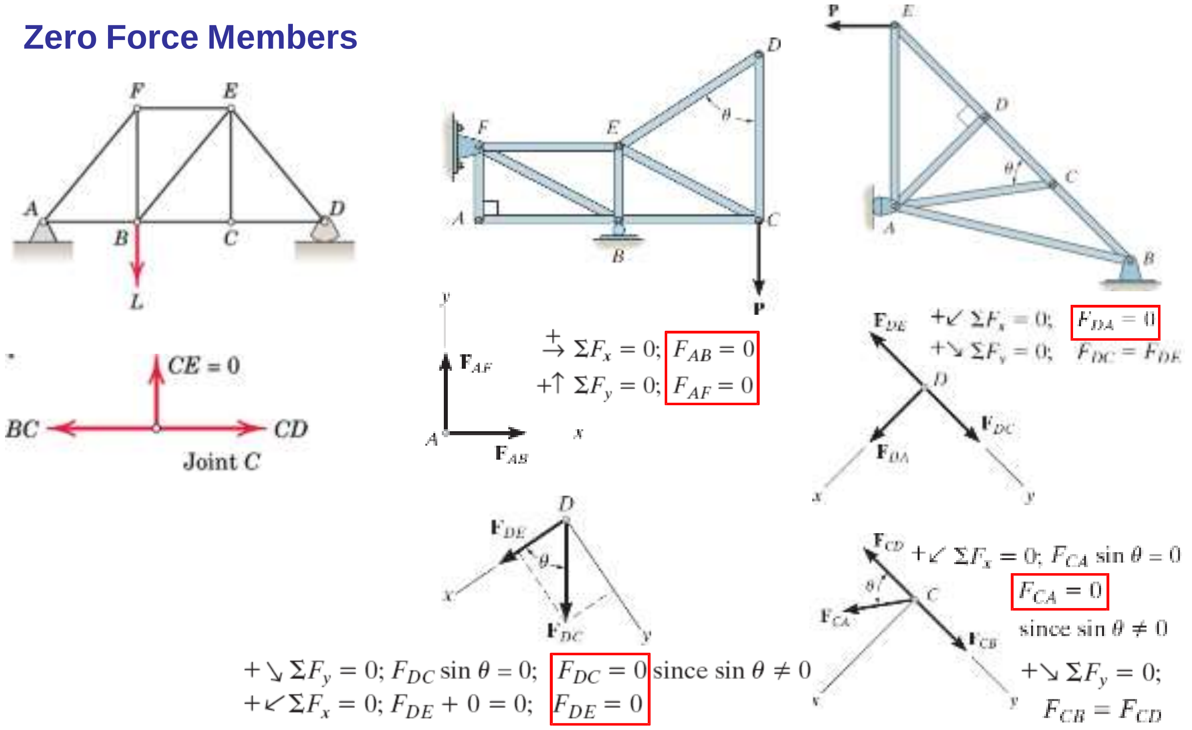
Structural Analysis: Plane Truss

Why to Provide Redundant Members?

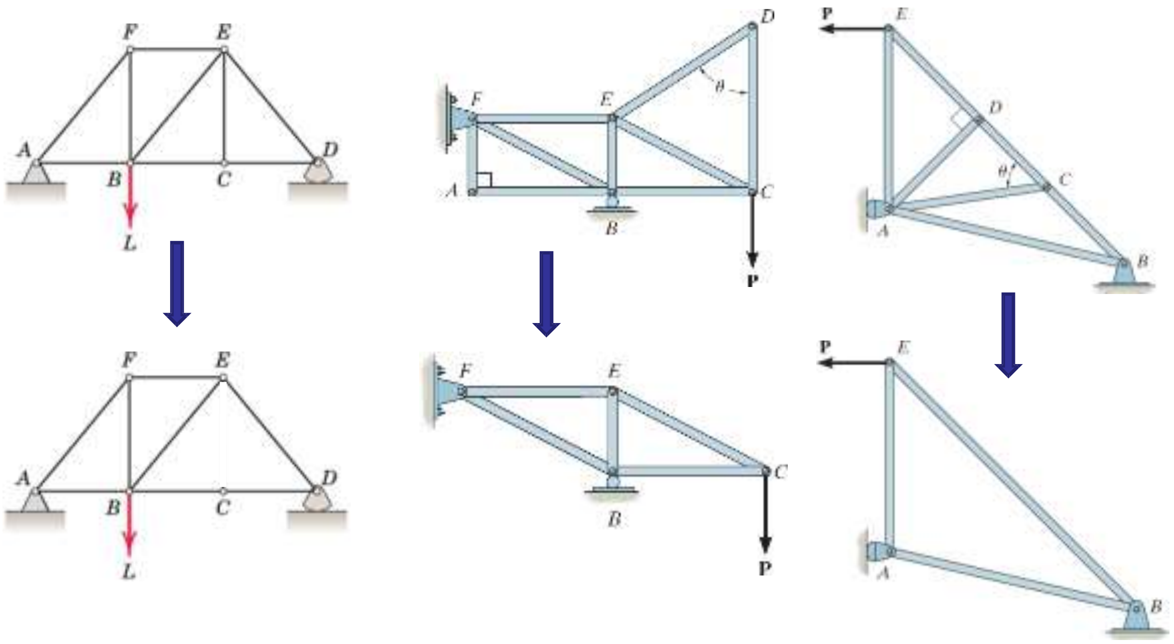
- To maintain alignment of two members during construction
- To increase stability during construction
- To maintain stability during loading (Ex: to prevent buckling of compression members)
- To provide support if the applied loading is changed
- To act as backup members in case some members fail or require strengthening
- Analysis is difficult but possible

Structural Analysis: Plane Truss

Zero Force Members



Structural Analysis: Plane Truss

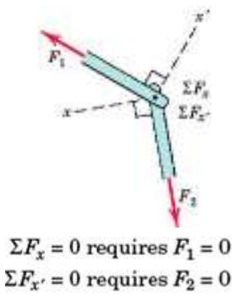


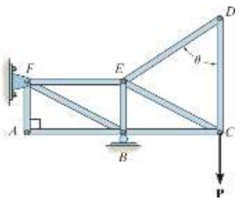
Structural Analysis: Plane Truss

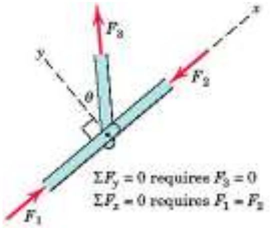
Zero Force Members: Conditions

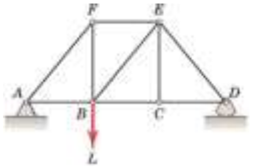
If only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero force members

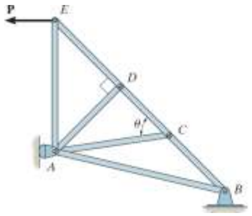
If three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint







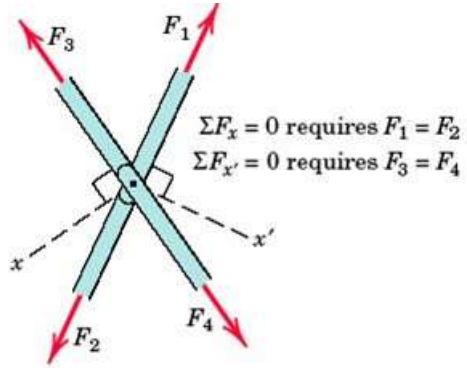




Structural Analysis: Plane Truss

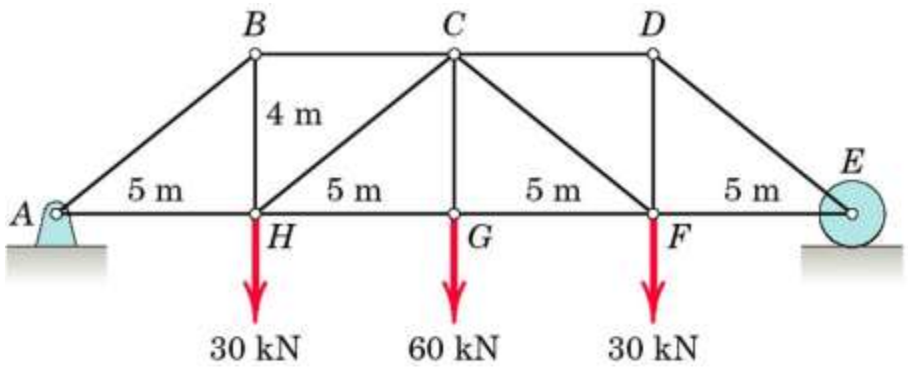
Special Condition

When two pairs of collinear members are joined as shown in figure,
the forces in each pair must be equal and opposite.



Method of Joints: Example

Determine the force in each member of the loaded truss by Method of Joints.



Is the truss statically determinant externally?

Yes

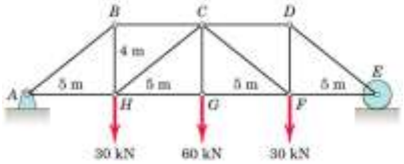
Is the truss statically determinant internally?

Yes

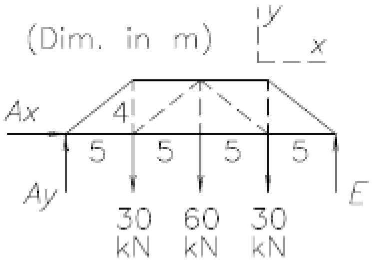
Are there any Zero Force Members in the truss?

No

Method of Joints: Example



Solution



As a whole: $\Sigma F_x = 0 \Rightarrow A_x = 0$

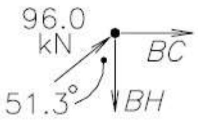
$A_y = E = 60$ kN by

$\Sigma F_y = 0$ and symmetry.

Joint A: $\left(\theta = \tan^{-1}(4/5) = 38.7°\right)$

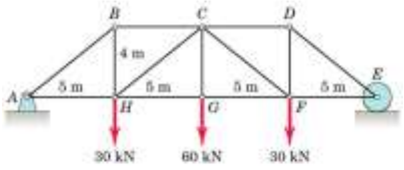
$$\begin{cases} \Sigma F_y = 0 : 60 - AB\sin\theta = 0, \ \underline{AB = 96.0 \text{ kN } C} \\ \Sigma F_x = 0 : AH - 96.0\cos\theta, \ \underline{AH = 75 \text{ kN } T} \end{cases}$$

Joint B:

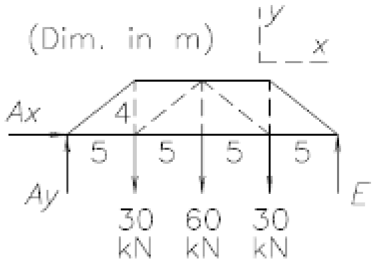


$$\begin{cases} \Sigma F_x = 0 : BC + 96.0\sin 51.3° = 0, \ \underline{BC = -75 \text{ kN } (C)} \\ \Sigma F_y = 0 : -BH + 96.0\cos 51.3° = 0, \ \underline{BH = 60 \text{ kN } T} \end{cases}$$

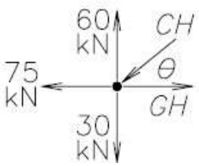
Method of Joints: Example



Solution

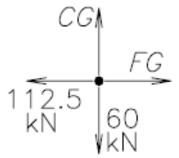


Joint *H*:



$$\begin{cases} \Sigma F_y = 0: -CH\sin\theta + 30 = 0, \ \underline{CH = 48.0 \text{ kN } C} \\ \Sigma F_x = 0: 48.0\cos\theta + GH - 75 = 0, \ \underline{GH = 112.5 \text{ kN } T} \end{cases}$$

Joint *G*:



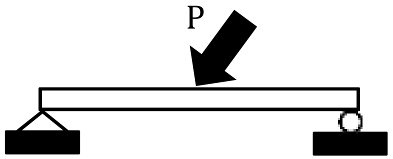
$$\Sigma F_y = 0 \Rightarrow CG = 60 \text{ kN } T$$

By symmetry:

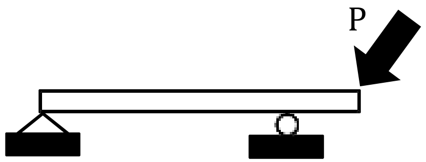
$FG = 112.5$ kN T, $CF = 48.0$ kN C

$CD = 75$ kN C, $DF = 60$ kN T

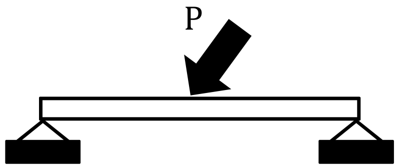
$\underline{EF = 75 \text{ kN } T, \ DE = 96.0 \text{ kN } C}$



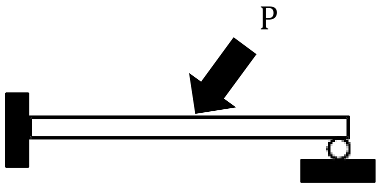
DETERMINATE



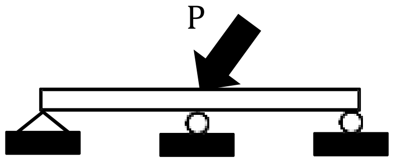
DETERMINATE



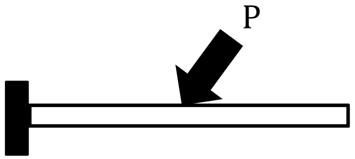
INDETERMINATE



INDETERMINATE

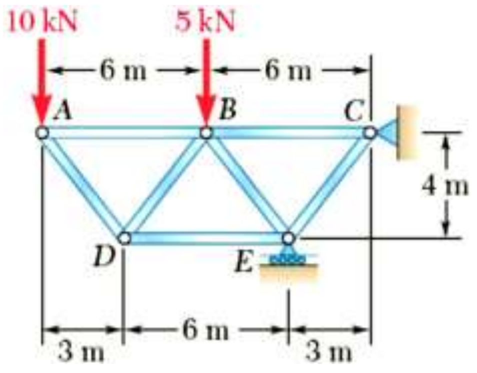


INDETERMINATE



DETERMINATE

Example

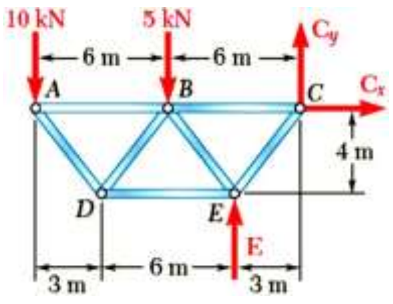


Using the method of joints, determine the force in each member of the truss.

SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.
- Joint *A* is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.
- In succession, determine unknown member forces at joints *D*, *B*, and *E* from joint equilibrium requirements.
- All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.

Example



SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at E and C.

$$\sum M_C = 0$$
$$= (10 \text{ kN})(12 \text{ m}) + (5 \text{ kN})(6 \text{ m}) - E(3 \text{ m})$$

$$E = 50 \text{kN} \uparrow$$

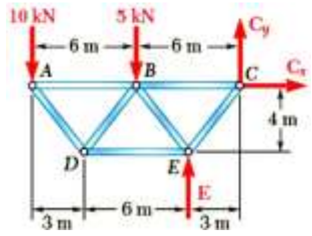
$$\sum F_x = 0 = C_x$$

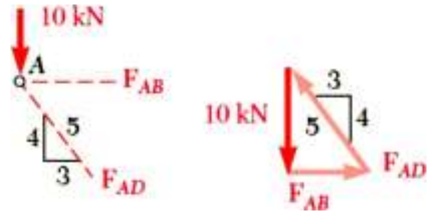
$$C_x = 0$$

$$\sum F_y = 0 = -10 \text{kN} - 5 \text{ kN} + 50 \text{ kN} + C_y$$

$$C_y = 35 \text{ kN} \downarrow$$

Example

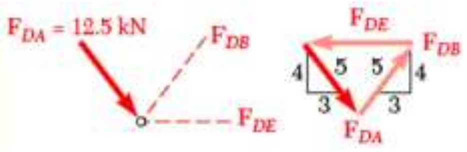




- Joint *A* is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.

$$\frac{10\,\text{kN}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$F_{AB} = 7.5\,\text{kN} \;\; T$$
$$F_{AD} = 12.5\,\text{kN} \;\; C$$

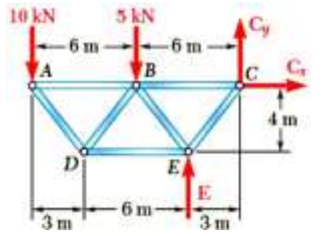


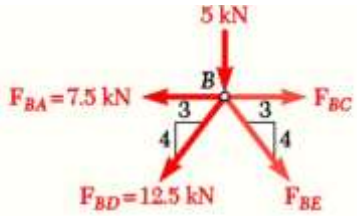
- There are now only two unknown member forces at joint D.

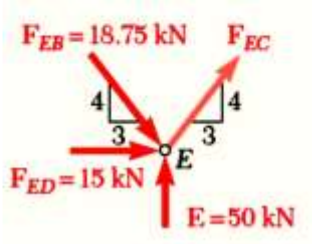
$$F_{DB} = F_{DA}$$
$$F_{DE} = 2\left(\tfrac{3}{5}\right)F_{DA}$$

$$F_{DB} = 12.5\,\text{kN} \;\; T$$
$$F_{DE} = 15\,\text{kN} \;\; C$$

Example







- There are now only two unknown member forces at joint B. Assume both are in tension.

$$\sum F_y = 0 = -5\text{kN} - \tfrac{4}{5}(12\text{kN}) - \tfrac{4}{5}F_{BE}$$

$$F_{BE} = -18.75 \text{ kN} \qquad \boxed{F_{BE} = 18.75 \text{ kN } C}$$

$$\sum F_x = 0 = F_{BC} - 7.5\text{kN} - \tfrac{3}{5}(12.5\text{kN}) - \tfrac{3}{5}(18.75)$$

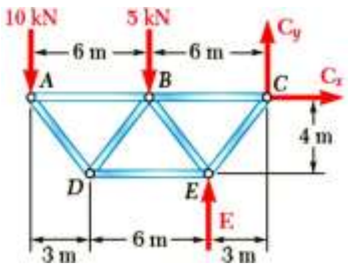
$$F_{BC} = +26.25 \text{ kN} \qquad \boxed{F_{BC} = 26.25 \text{ kN } T}$$

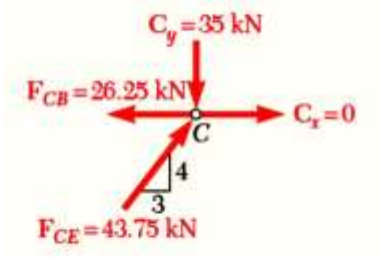
- There is one unknown member force at joint *E*. Assume the member is in tension.

$$\sum F_x = 0 = \tfrac{3}{5}F_{EC} + 15\text{kN} + \tfrac{3}{5}(18.75\text{kN})$$

$$F_{EC} = -43.75 \text{ kN} \qquad \boxed{F_{EC} = 43.75 \text{ kN } C}$$

Example





- All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.

$$\sum F_x = -26.25 + \tfrac{3}{5}(43.75) = 0 \quad \text{(checks)}$$

$$\sum F_y = -35 + \tfrac{4}{5}(43.75) = 0 \quad \text{(checks)}$$

Thanks
For
Your Attention